

# **TACTICAL AND OPERATIONAL PLANNING FOR PER-SEAT, ON-DEMAND AIR TRANSPORTATION**

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# TACTICAL AND OPERATIONAL PLANNING FOR PER-SEAT, ON-DEMAND AIR TRANSPORTATION

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*To my parents.*

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# TABLE OF CONTENTS

DEDICATION . . . . .	iii
ACKNOWLEDGEMENTS . . . . .	iv
LIST OF TABLES . . . . .	vii
LIST OF FIGURES . . . . .	viii
SUMMARY . . . . .	ix
I INTRODUCTION . . . . .	1
1.1 Per-Seat, On-Demand Air Transportation . . . . .	3
1.2 Tactical Planning for PSOD Air Transportation . . . . .	4
1.3 Operational Planning for PSOD Air Transportation . . . . .	5
1.4 Thesis Outline and Contributions . . . . .	6
1.5 Literature Review . . . . .	12
II TACTICAL MAINTENANCE CAPACITY PLANNING . . . . .	17
2.1 Introduction . . . . .	17
2.2 Tactical Planning for Maintenance Capacity in the Growth Phase . . . . .	19
2.2.1 History-Independent Tactical Capacity Planning Problem . . . . .	21
2.2.2 Solution Approach . . . . .	23
2.2.3 Computational Results . . . . .	26
2.3 Tactical Planning for Steady State Maintenance Capacity . . . . .	32
2.3.1 Properties of the maintenance intervals . . . . .	34
2.3.2 Generating a Feasible Maintenance Schedule . . . . .	35
2.3.3 Determining optimal and long run capacities . . . . .	40
III OPERATIONAL PLANNING FOR SCHEDULED MAINTENANCE . . . . .	43
3.1 Introduction . . . . .	43
3.2 Solution Approach . . . . .	44
3.2.1 Maintenance Decision Phase . . . . .	45

3.2.2	Smoothing phase . . . . .	50
3.2.3	Computational Results . . . . .	52
3.3	Integration with Flight Scheduling . . . . .	58
3.3.1	Case Study . . . . .	60
IV	BASE LOCATION AND FLEET ALLOCATION . . . . .	65
4.1	Introduction . . . . .	65
4.2	Problem Description . . . . .	67
4.3	Basic approach . . . . .	69
4.3.1	Solution Approach . . . . .	73
4.3.2	Computational Results . . . . .	79
4.4	Integrated Approach . . . . .	89
4.4.1	Solution Approach . . . . .	93
4.4.2	Computational Results . . . . .	94
V	CONCLUSIONS AND FUTURE RESEARCH . . . . .	98
	REFERENCES . . . . .	101

## LIST OF TABLES

2.1	Summary results for the tactical capacity planning problem . . . . .	31
3.1	Summary results for the operational maintenance planning problem .	55
3.2	Summary results for the simulated case study . . . . .	64
4.1	Optimality gap estimates for the activity-based demand scheme . . .	81
4.2	Optimality gap estimates for the imbalance-based demand scheme . .	82
4.3	Base location and fleet allocation decisions for each time period for the activity-based demand scheme . . . . .	83
4.4	Base location and fleet allocation decisions for each time period for the imbalance-based demand scheme . . . . .	83
4.5	Average acceptance rate for the transportation requests with the basic approach . . . . .	86
4.6	Average of the total daily flying time with the basic approach . . . .	86
4.7	Base location and fleet allocation decisions for each time period for the activity-based demand scheme when all decisions are made simultaneously . . . . .	87
4.8	Base location and fleet allocation decisions for each time period for the imbalance-based demand scheme when all decisions are made simultaneously . . . . .	87
4.9	Summary statistics for flight scheduling with the basic approach when decisions for all time periods are made simultaneously . . . . .	88
4.10	Summary of the average acceptance rate and average daily flying time for the basic approach . . . . .	89
4.11	Base location and fleet allocation decisions for each time period made with the integrated approach . . . . .	95
4.12	Summary statistics for flight scheduling with the integrated approach	97
4.13	Summary of the average acceptance rate and average daily flying time for both approaches . . . . .	97

## LIST OF FIGURES

2.1	Allowable maintenance intervals for a jet . . . . .	19
2.2	Optimization-based local search progress within half an hour . . . . .	28
2.3	Optimization-based local search progress between half an hour and two hours . . . . .	29
2.4	Smooth Maintenance Schedule $P_k$ for $k \leq \bar{k}$ . . . . .	35
2.5	Smooth Maintenance Schedule $P_k$ for $k > \bar{k}$ . . . . .	36
2.6	Example Maintenance Schedule . . . . .	41
3.1	Impact of large fluctuations in the number of jets with different accumulated flying hours . . . . .	45
3.2	Part of the network for a single jet . . . . .	49
3.3	Maintenance chart for instance (288J,2D) . . . . .	54
3.4	Maintenance chart for instance (288J,2M) . . . . .	56
3.5	Histogram of the accumulated flying hours for instance (288J,2D) . . . . .	57
3.6	Histogram of the accumulated flying hours for instance (288J,2M) . . . . .	58
3.7	Maintenance chart for the simulated case study . . . . .	62
3.8	Histogram of the accumulated flying hours for the simulated case study . . . . .	63
4.1	Different ways of aggregating transportation requests with at most one intermediate stop . . . . .	71
4.2	Finding the imbalance between incoming and outgoing requests at an airport . . . . .	73
4.3	Example depicting the difference between the demand schemes . . . . .	73
4.4	Bases opened with the activity-based demand scheme . . . . .	84
4.5	Bases opened with the imbalance-based demand scheme . . . . .	85
4.6	Bases opened with the activity-based demand scheme when decisions for all time periods made simultaneously . . . . .	89
4.7	Bases opened with the imbalance-based demand scheme when decisions for all time periods made simultaneously . . . . .	90
4.8	Bases opened with the integrated approach . . . . .	96



## SUMMARY

Advances in aviation technology including the development of relatively cheap, very light jets and the possibility of free-flight have led to the realization of a per-seat, on-demand (PSOD) air transportation business that operates without a published flight schedule. This thesis addresses two fundamental planning problems motivated by the operations of PSOD air transportation. The first problem focuses on the scheduled maintenance of the fleet that has to be done periodically for safety and efficiency. The second problem is concerned with selecting locations for bases and determining how many jets to allocate to each base where bases are airports with hangar space to keep jets overnight. These decisions have a significant impact on the ability of the business to accommodate transportation requests and also to satisfy these requests efficiently.

In the first part of the thesis, we study tactical decision making for scheduled maintenance planning that determines the daily maintenance capacities, i.e the maximum number of jets that can be maintained on a day. These decisions are made for two operating conditions: a growth phase where jets are introduced gradually into the system and steady state where the fleet size is constant. We model the tactical maintenance capacity planning during the growth phase as an integer program and develop an optimization-based local search to solve the problem. We present a computational study that investigates the impact of the frequency in which jets are introduced into the system on the maintenance capacity. The results illustrate that around 14% less overall capacity is needed when jets are introduced more frequently in smaller batches. Tactical planning for scheduled maintenance of PSOD air transportation in the steady state is NP-hard. We analyze a special case of this

problem for which we can determine the optimal and the long run capacities with a pseudo-polynomial time algorithm.

In the second part of the thesis, we address the operational planning for scheduled maintenance. Operational level planning is concerned with assigning itineraries to jets and determining the specific jets to be scheduled for maintenance on a daily basis given a certain maintenance capacity. We present a solution methodology that employs a look-ahead approach to consider the impact of our current decisions on the future and decomposes the problem exploiting the differences between jets with respect to their proximity to the next maintenance. The methodology can effectively schedule maintenance of 480 jets over a two year planning horizon where the decisions for a single day can be made on average within 12 seconds. Furthermore, an average capacity usage rate of 96% together with less than 1% infeasible maintenance indicate a good match between the capacities set at the tactical and the operational maintenance needs. We further develop an integrated framework in order to capture the interaction between the operational level maintenance decisions and flight scheduling. A simulated case study for the operations of a PSOD air transportation provider, DayJet Corporation, demonstrates that only 6% of the maintenance activities have to be delayed by on average one day to accommodate the requirements of the flight scheduling.

In the third and final part of the thesis, we present the tactical level base location and fleet allocation problem. As PSOD air transportation experiences changes in travel demand and fleet size, decisions regarding where to open new bases and how to allocate the number of jets amongst these bases are made. We first present a solution approach in which the information about travel demand (in the form of transportation requests) and flight scheduling is used in a traditional facility location problem. We next develop a model that works directly with transportation requests and integrates a simplified version of flight scheduling with the base location and fleet

allocation decisions. Thus, the information about travel demand and flight scheduling is captured in more detail compared to the traditional facility location problem. The results of our computational study illustrate that an average of 2% increase in the acceptance rate for transportation requests, and an average of 4% decrease in the average daily flying time can be achieved when travel demand and flight scheduling are captured in more detail while making base location and fleet allocation decisions.

# CHAPTER I

## INTRODUCTION

The competitive nature of modern day business environments makes carefully planned tactical and operational decisions more compelling than ever. *Tactical planning* involves determining operational settings and allocating resources over a relatively long planning horizon whereas *operational planning* is concerned with day-to-day decisions. For example, in freight transportation tactical planning typically includes the design of the service network and work allocation among terminals while routing and dispatching of vehicles and scheduling of maintenance activities are performed at the operational level (Crainic and Laporte [6]). Similarly, in production planning capacity is allocated to different product families and workforce availability is determined at the tactical level while the amount of each product type to be produced on a daily basis is established at the operational level (Bitran et al. [5]). Forestry applications provide another example where tactical planning sets the target harvest levels for macro stands and the specific stands to be harvested are selected at the operational level (Martell et al. [25]). As the higher level tactical planning spans a longer time horizon, certain simplifying assumptions are made regarding the operations and data is aggregated to ensure tractability. In freight transportation routes or loads are typically consolidated, in production planning product types are aggregated into product families, and macro stands are formed in forestry applications. In addition to providing tractability, such aggregation makes tactical level decisions less sensitive to variations in the input data.

There is a high level of interaction between the decision making processes of tactical and operational levels. On one hand, tactical planning determines the goals and

the resource allocations that guide the decisions at the operational level. Conversely, effective resource allocations cannot be made without knowing the requirements of the day-to-day decisions. An integrated approach that combines these two levels is appealing due to this interdependence. Examples of such integrated approaches are found in Agarwal and Ergun [1] where tactical level ship scheduling and operational level cargo routing are handled simultaneously for liner shipping, in Weintraub et al. [47] where tactical harvest planning and operational harvest scheduling are considered within a single model for forestry, and in Goetschalckx et al. [15] where it is demonstrated that savings can be achieved by integrating the design of supply chain networks with the production-distribution decisions. However, integrated models tend to be very large for most practical applications and thus, operations cannot be analyzed in detail to provide tractability. Furthermore, from a managerial standpoint these two levels of planning might be handled by different decision makers. Thus, most common solution methodologies use hierarchical approaches where tactical and operational planning are performed sequentially. Decisions made at the tactical level using necessary information about the operations become an input to the operational level. Solutions that are consistent at both planning levels can be obtained provided that the information used at the tactical level is fairly accurate and suitable aggregation and disaggregation schemes are chosen. Examples of studies where hierarchical approaches are successfully applied can be found in Saad [35] for production planning and in Weintraub and Cholakay [46] for forest planning.

This thesis addresses two tactical and operational planning problems motivated by the operations of per-seat, on-demand (PSOD) air transportation companies such as DayJet Corporation ([www.dayjet.com](http://www.dayjet.com)), MyJet ([www.myjetindia.com](http://www.myjetindia.com)) and Fly-Miwok ([www.flymiwok.com](http://www.flymiwok.com)). The first problem focuses on planning the scheduled maintenance of the fleet as mandated by the Federal Aviation Administration (FAA)

and manufacturer regulations to ensure safety. Tactical decisions related to maintenance capacity are made while the questions pertaining to which jets to maintain on a daily basis are answered at the operational level. The second problem is concerned with planning base location and fleet allocation. Bases are airports with hangar space to keep jets in storage during the night and where pilots are domiciled. At the tactical level, the airports that will be bases are selected and the number of jets to be allocated to each base is determined strategically to serve the demand as best as possible while the daily flight schedules for the jets are constructed at the operational level. Both problems deal with significant capital expenditures and have huge impacts on the ability of the business to generate profit.

### ***1.1 Per-Seat, On-Demand Air Transportation***

Advances in aviation technology including the development of relatively cheap, very light jets and the possibility of free-flight have led to the realization of a PSOD air transportation business that operates without a published flight schedule. In this model, travelers call a few days in advance to request transportation by providing their origin airport, destination airport, an earliest departure time from the origin and a latest arrival time at the destination. An online scheduling algorithm quickly determines whether a request can be accommodated. However, the flight schedules for a given day are not finalized until the night before when itineraries (sets of consecutive flight legs to be flown by a single jet) are constructed to accommodate all accepted requests using an off-line scheduling algorithm. Thus, on any given day only the itineraries for that day are known with certainty.

The PSOD model has several advantages to a business traveler over the alternative offered by a schedule-operated airline. Schedule-operated airlines fly mostly between heavily congested airports that have long security lines and their flights usually have connections through hub airports that might be far away from travelers' origins and

destinations. Together, these result in long travel times and it becomes impossible to complete a business trip without staying overnight away from home. Contrary to these traditional norms of air travel, PSOD air transportation offers travel from and to small, regional airports that are less congested. The significantly shorter travel times eliminate the need for overnight stay while providing more pleasant trips. Furthermore, travelers have more flexibility as the flight plans are formed around their schedules rather than the other way around.

## ***1.2 Tactical Planning for PSOD Air Transportation***

As PSOD air transportation is a new business model and the companies offering PSOD air transportation are in the early stages of their development, they face a number of challenging tactical decision problems. One of these problems is concerned with the purchase of new jets and timing their introduction into the fleet. One has to consider the trade-offs between the required capital expenditure for a new jet and the opportunity cost of losing business due to rejected requests that could not be accommodated given the current size of the fleet. Determining the size of the pilot pool required to operate these jets and hiring qualified pilots are also integral to the tactical decision making. Another tactical decision concerns the pricing of a flight between two locations given the size of the time intervals for departure and arrival.

This thesis analyzes specifically two tactical decision problems: determining the maintenance capacity, and locating bases and allocating the jets in the fleet to the bases. Jets have to be maintained systematically in order to ensure safe continuation of operations. To this end, determining the maintenance capacity is a fundamental tactical decision to be made. Tactical maintenance capacity planning needs to analyze the trade-off between setting the maintenance capacity high to ensure timeliness of maintenance and the cost of this capacity to achieve safe and efficient operations. A similar trade-off has to be made in the base location and fleet allocation problem.

Although the business operates between a large number of airports, only a small portion of these airports are to be selected as bases due to the large investment associated with the addition of a new base. On the other hand, opening more bases gives the opportunity to have jets ready to pick up passengers at a larger number of locations and thus, increases the revenue as well as decreasing the operational costs.

### ***1.3 Operational Planning for PSOD Air Transportation***

Day-to-day operations of PSOD air transportation are driven by some fundamental operational planning decisions. Flight scheduling is one of the most challenging day-to-day decisions of this business which operates in a dynamic environment. It involves constructing cost-efficient itineraries under certain conditions. As mentioned before, an itinerary should start and end at the same base in order to provide better quality of life for the pilots. Furthermore, each flight leg in an itinerary should be feasible with respect to the seating capacity and total weight limitations. As a significant portion of the operational costs comes from fuel related expenses, the aim is to minimize the total travel time including both revenue and deadhead legs while constructing these itineraries. Flight scheduling affects the profitability of the business as well as the quality of service and requires employment of optimization techniques (Espinoza et al. [12, 13] Engineer et al. [11]). Inputs to operational level flight scheduling are the decisions made at the tactical level for the pricing of the potential flight legs as well as the base location and fleet allocation. This thesis addresses the interactions between the operational level flight scheduling and the tactical decisions of locating bases and allocating jets.

In order to avoid frequent breakdowns and provide safe operations, the jets in the fleet have to undergo periodic maintenance. This thesis considers the day-to-day decisions related to maintenance planning and their interactions with the tactical



level decisions determining the maintenance capacity. Maintenance affects the availability of jets to fly and they must be planned taking other daily operations of the business into account. In order to capture this, the interaction between operational maintenance planning and daily flight scheduling is also analyzed in this thesis.

## ***1.4 Thesis Outline and Contributions***

Two fundamental planning problems of PSOD air transportation are analyzed in this thesis: (i) scheduled maintenance planning and (ii) base location and fleet allocation. The organization of the chapters is as follows. Tactical and operational level scheduled maintenance planning problems are discussed in Chapter 2 and Chapter 3, respectively. Chapter 4 addresses the tactical planning problem of base location and fleet allocation and its interaction with operational level flight scheduling. Finally, Chapter 5 presents conclusions and future research directions.

Due to physical space limitations or the availability of labor, there is an upper bound on the number of jets that can be maintained on a given day. This is referred to as the maintenance capacity. Tactical level decision making for scheduled maintenance planning determines the maintenance capacity. In Chapter 2, these decisions are made for two operating conditions: (i) a growth phase where jets are introduced gradually into the system and (ii) a steady state where the fleet size is constant.

During the growth phase, the increasing fleet size leads to a need to increase the maintenance capacity over time. Thus, the tactical decisions for maintenance capacity concern when and how much to increase the capacity. We model this problem as an integer program and provide valid inequalities to strengthen the formulation and improve the lower bounds. Furthermore, we develop an optimization-based local search to find good solutions quickly. The computational study investigates the impact of the frequency in which jets are introduced into the fleet. The results illustrate that up to 14% less overall capacity is sufficient when jets are introduced more frequently

in smaller batches as it is possible to distribute the maintenance activities of the jets more evenly over time.

In the steady state, we need to determine a single level of maintenance capacity as the fleet size and hence the maintenance workload does not change over time. The problem of determining the optimal capacity for a periodic scheduling problem where a set of machines need to be maintained at constant intervals is shown to be NP-hard in Mok et al. [27]. Tactical planning for scheduled maintenance of PSOD air transportation is a generalization of the aforementioned problem and thus, is NP-hard. In Chapter 2, we analyze a special case of tactical planning for scheduled maintenance of PSOD air transportation where the optimal and the long run capacities can be determined with a pseudo-polynomial time algorithm that provides a feasible maintenance schedule.

The main contributions of this thesis to tactical maintenance capacity planning can be summarized as follows:

- We have studied capacity planning for periodic scheduling where the number of tasks to be scheduled grows over time, which is an area that has not been previously addressed in the literature. More specifically,
  - we have developed an integer program to model the problem,
  - we have developed an optimization-based local search that finds high-quality solutions quickly, and
  - we have analyzed the impact of a jet introduction schedule on the required maintenance capacity and have shown that by carefully introducing jets into the system maintenance capacity costs can be reduced significantly.
- We have provided a pseudo-polynomial time algorithm to determine the optimal and the long run capacities for the special case of steady state maintenance capacity planning for PSOD air transportation in which all jets are introduced

into the system in batches of equal size on consecutive days at the start of operations.

Operational level maintenance planning, which is presented in Chapter 3, is concerned with assigning itineraries to jets and determining the specific jets to be scheduled for maintenance on a daily basis given a certain maintenance capacity. We employ a look-ahead approach in order to consider the impact of our current decisions on the future and solve a  $k$ -day problem on each day. Furthermore, we decompose the operational maintenance planning problem into two phases, where we make decisions for critical jets, i.e. jets that can be maintained in the next  $k$  days, in Phase I and decisions for non-critical jets in Phase II. The methodology can effectively schedule maintenance of 480 jets over a two year planning horizon where the decisions for a single day can be made on average within 12 seconds. Furthermore, an average capacity usage rate of 96% together with less than 1% infeasible maintenance indicate a good match between the capacities set at the tactical and the operational maintenance needs.

There is a strong interaction between the operational level maintenance decisions and flight scheduling. On one hand, maintenance affects the availability of jets as they are removed from circulation for the duration of their maintenance and thus, it might not be possible to accept some requests. Conversely, it might be necessary to revise the maintenance decisions if the already accepted requests cannot be accommodated with the changes induced by maintenance. In order to capture this interaction, we develop a framework in which operational maintenance planning and daily flight scheduling can get feedback from each other. The framework is tested in a simulation environment where transportation requests are generated using an agent-based model, and flight scheduling and operational maintenance planning are performed in real-time. The results of the case study demonstrates that only 6% of the maintenance activities have to be delayed by on average 1 day to accommodate

the requirements of the flight scheduling.

The main contributions of this thesis to operational maintenance planning can be summarized as follows:

- We have developed an optimization-based decision support tool for operational maintenance planning that can effectively determine the maintenance schedule together with the itinerary assignments of a fleet of upwards of several hundred jets during a two and a half year planning horizon.
- We have built a framework that integrates maintenance decisions and flight scheduling. Computational results indicate that high-quality solutions to real life instances can be achieved using this framework.

The tactical planning problem of base location and fleet allocation is presented in Chapter 4. As PSOD air transportation experiences changes in travel demand and fleet size, decisions regarding where to open new bases and how to allocate the number of jets amongst these bases are made. In order to solve the multi-period base location and fleet allocation problem, we need information about travel demand. We consider two possibilities for our knowledge of this information. Firstly, we assume that travel demand information is available for the entire planning horizon and thus, we are free to make decisions for all time periods simultaneously. Alternatively, a more realistic setting is to assume that travel demand information is updated at regular intervals during the planning horizon. With this assumption, we determine the base location and fleet allocation decisions periodically making the best decisions given the information available up to a certain time point.

While solving the base location and fleet allocation problem, we use two approaches that capture the information about travel demand and operational flight scheduling with different levels of detail. Our aim is to investigate (i) the impact of

base location and fleet allocation decisions on operational flight scheduling, specifically the acceptance rate for the transportation requests and the average daily flying time, and (ii) the impact of considering different levels of detail about travel demand and flight scheduling on the quality of this decision making.

The first approach uses a traditional facility location model where the information about travel demand and flight scheduling is captured in an input that takes on the form of the number of jets demanded at each airport. This demand input is found using two main schemes to distribute the total number of jets to the airports. The first one is referred to as the *activity based* scheme and gives more weight to airports with a larger total number of outgoing requests. The second one gives more weight to the airports with a larger imbalance of incoming and outgoing requests during the day to avoid deadheads and is referred to as the *imbalance based* scheme. Since the travel demand and thus the number of jets demanded at each airport is not known with certainty, the resulting model is a two-stage stochastic program. The first stage decisions are related to the base location and fleet allocation whereas the second stage decisions determine how the demand of an airport is satisfied once the demand is known. The sample average approximation (SAA) method is used to solve this stochastic facility location problem. In our computational study, we first solve the base location and fleet allocation problem with the assumption of complete travel demand information for the entire planning horizon and make the decisions for all time periods simultaneously. We next solve the problem assuming that travel demand information is updated at certain time points and the decisions are made periodically. The results of the computational study indicate that the quality of the solutions obtained by solving the problem periodically are comparable to the ones obtained by solving it at once.

In the second approach, we develop a model that works directly with transportation requests and integrates a simplified version of flight scheduling with the base

location and fleet allocation decisions. That is, the base location and fleet allocation decisions are made while determining the decisions regarding the routing of the transportation requests and the jets. Including more details in the model increases the complexity of the problem and it becomes impossible to make decisions for all time periods simultaneously. Thus, we make the base location and fleet allocation decisions only periodically while using this approach. The decisions at each time period are made as follows. Since transportation requests are not known with certainty, we solve the problem for several different realizations of daily transportation requests in order to account for the variability in the data. The different solutions obtained are then merged into one solution that we deem to be the best using a two-step optimization method. In our computational study, we compare the base location and fleet allocation decisions made using the second approach to the ones that are made periodically using the first approach. The results indicate that an average of 2% increase in the transportation request acceptance rate and an average of 4% decrease in the average daily flying time can be achieved when travel demand and flight scheduling are captured in more detail while making the base location and fleet allocation decisions.

The main contributions of this thesis to tactical planning of base location and fleet allocation can be summarized as follows:

- We have developed two approaches to incorporate the information about travel demand and operational flight scheduling into the tactical decision making of base location and fleet allocation that differ in the amount of detail considered.
- We have conducted computational studies that demonstrate the quality of the base location and fleet allocation decisions obtained by our models in terms of two important performance measures of PSOD air transportation: the acceptance rate for the transportation requests and the average daily flying time.

## 1.5 Literature Review

One of the most important tasks PSOD air transportation faces is scheduling the itineraries for accepted transportation requests to be flown on a given day. Espinoza et al. [12] presents an integer multi-commodity network flow model with side constraints for constructing the itineraries. They solve small to medium size instances by developing techniques to control the size of the network and to strengthen the linear programming relaxation. Espinoza et al. [13] embeds this methodology in a parallelized local search framework to produce high-quality solutions efficiently for large-scale real-life instances. To measure the quality of the flight schedules constructed, Engineer et al. uses a column generation formulation to compute tight bounds on the optimal objective value.

Scheduled maintenance planning in which maintenance has to be performed at regular intervals is a periodic scheduling problem. Periodic scheduling is a well researched area with a broad range of applications. These include scheduling periodic real-time tasks on computer processors (Dhall and Liu [9]), data dissemination in teletext and wireless systems via broadcast disks (Kenyon et al. [21]), multi-item replenishment of stock (Anily et al. [2], Bar-Noy et al. [4]) and so on. However, most of the literature is on the operational planning problem for a given capacity while there are only a few studies that address the capacity planning problem. Furthermore, to the best of our knowledge, there is no study in the literature that addresses capacity planning for periodic scheduling where the number of tasks to be scheduled increases over time.

The literature on capacity planning for periodic scheduling is mostly theoretical. The early work of Dhall and Liu [9] addresses the problem of finding the minimum number of processors to schedule periodic real-time tasks. They present worst-case results on two heuristic algorithms to divide the set of tasks into groups such that the tasks in each group can be feasibly scheduled on a single processor. However, their

problem differs from ours as they consider preemptive scheduling. A special case where the time interval between two occurrences of a periodic activity is constant is analyzed in Park and Yun [32], Mok et al. [27] and Wei and Liu [45]. Park and Yun [32] presents an approach to partition their proposed integer programming formulation into smaller independent ones based on the chinese remainder theorem. A polynomial-time testable condition that guarantees the existence of a feasible schedule with a given number of servers is given in Wei and Liu [45]. However, the problem is later shown to be NP-hard in Mok et al. [27] where two special cases are analyzed for which they present polynomial time algorithms.

Starting with the early work of Wagner et al. [44], several studies address the operational scheduled maintenance planning problem. Some of these studies, such as Anily et al. [2], [3], Bar-Noy et al. [4], Grigoriev et al. [17] and Kenyon et al. [21] address a problem in which the operating cost of a machine increases with time since its last maintenance. The aim is to minimize the long-run average cost per time. This problem differs from ours as there is no explicit time until the next maintenance.

Scheduled maintenance planning problems similar to ours have been studied in Mattila and Virtanen [26] for scheduling the periodic maintenance of a fleet of fighter aircraft, in Haghani and Shafahi [18] for scheduling bus maintenance activities, in Deris et al. [8] for ship maintenance scheduling, and in Kralj and Petrovic [23] for periodic maintenance scheduling of thermal units in electric power systems. The problems addressed in these studies do not have an additional assignment component to determine the daily use of vehicles/machines as it is constant and known. However, in our problem the flying time of the itineraries is not constant and thus, it is necessary to model the decisions pertaining to the assignment of itineraries to specific jets.

Gopalan and Talluri [16] review models and solution approaches for maintenance planning in schedule operated airlines. These models and solution approaches are quite different from ours. First, maintenance capacity is never considered to be a



binding constraint in these studies. However, we try to allocate the lowest possible maintenance capacity and thus, capacity is a tight constraint most of the time. Furthermore, in schedule operated airline operations, the timing between successive maintenance activities is usually set to be 3 or 4 days and the solution approaches are geared towards finding a feasible routing into a maintenance facility within these days. These solution approaches are not applicable to our problem due to the uncertainty of the itineraries. Sriram and Haghani [41] presents a formulation based on accumulated flying hours for schedule operated airlines. However, they do not develop a solution procedure for solving the problem with this formulation as an exact solution cannot be obtained in reasonable computation time due to the large size of the problem.

Location problems have been studied extensively due to their wide range of applications in planning for both private and public sectors. Due to the computational complexity, most research in this area has been limited to deterministic single-period problems where all inputs have known, constant values and the outputs are one time decisions. These problems have different objective functions and operating characteristics depending on the application area. For example, the objective of locating fire stations is to cover all potential customers using the minimum number of facilities such that the response time is within a certain limit whereas the objective of locating production facilities in a supply chain is to minimize the weighted distance between these facilities and the demand nodes. A general overview of such problems is given in reviews by ReVelle et al. [33], Daskin [7], Francis et al. [14] and Drezner [10].

Deterministic single-period location problems cannot address the uncertainties inherent in making real-world decisions. These uncertainties arise in two ways: (i) the uncertainty related to planning for an extended time horizon which is handled by multi-period models, and (ii) the uncertainty due to limited knowledge of input parameters which is handled by stochastic models. Owen and Daskin [31] gives an

overview of studies that address either the multi-period or stochastic characteristics of the location problems in order to overcome these limitations.

In one of our solution approaches for base location and fleet allocation, we analyze an extension of basic location models. Specifically, we address a fixed charge capacitated facility location problem where the demand of a certain location can be satisfied from several supply points and the selection of facility sizes is endogenous. A fixed charge facility location model that incorporates the endogenous selections of facility sizes in a single-period model is studied in Sankaran and Raghavan [36] and Mukundan and Daskin [28]. Scott [38] reviews location problems where the demand of a node can be satisfied from multiple supply points in a single-period setting whereas Scott [39] is one of the first studies to analyze such a problem in a multi-period setting without facility capacities where multiple facilities are located one at a time at equally spaced time epochs. They do not consider relocation of facilities over time. Wesolowsky and Truscott [48] extends the study made in Scott by allowing facilities to be relocated over time. Tapiero [42] further extends this study by including facility capacities. However, these capacities are fixed a priori instead of being endogenously determined.

Solution approaches for stochastic location problems include scenario planning approach, probabilistic approach and robust optimization. Mostly, either demand quantities or the travel times are taken to be uncertain. Snyder [40] reviews facility location under uncertainty. Romauch and Hartl [34] addresses a multi-period uncapacitated facility location problem with stochastic demands. They develop an SAA based solution approach to solve the problem. The results obtained with this approach are compared to the ones obtained by an exact solution approach based on stochastic dynamic programming for small instances. Santoso et al. [37] addresses supply chain network design under uncertainty which can be viewed as a capacitated facility location problem. They develop a solution approach that integrates SAA

method with Benders decomposition. The facility capacities in this study also are assumed to be given.

More recently, studies have been made to integrate tactical level facility location decisions with operational vehicle routing. Nagy and Salhi [29] review these studies that include multi-period or stochastic versions of the problem. The difference between our model in which we make base location and fleet allocation decisions together with routing of passengers and jets, and the studies referenced in [29] is that in our problem the routes change every day due to the on-demand nature of the business.

## CHAPTER II

### TACTICAL MAINTENANCE CAPACITY PLANNING

#### *2.1 Introduction*

An important daily operation for PSOD air transportation is the scheduled maintenance of the jets in the fleet. Scheduled maintenance is mandated by the Federal Aviation Administration (FAA) and manufacturer regulations to ensure safety and has to be done periodically. Unscheduled maintenance due to breakdowns or other unforeseen events is not included in this category. There are several types of scheduled maintenance. Some of these take a short amount of time and can be done at any airport. Others are more time consuming and are required to be done at a maintenance facility. In this thesis, we consider types of scheduled maintenance that are frequent enough to pose planning challenges and also require a visit to a maintenance facility. Furthermore, all maintenance activities are performed overnight at a single maintenance facility.

The types of scheduled maintenance considered in this thesis are driven by different attributes of the fleet such as accumulated flying hours and number of take-offs and landings since the last maintenance. For example, certain checks might have to be done every 300 hours while others have to take place after every 100 take-offs and landings. Since the analysis is the same for all these different attributes, we present the methodology in terms of accumulated flying hours.

Ideally, a jet should be maintained after accumulating a target number of flying hours ( $H$ ) since its last maintenance. However, in reality it is hard to achieve this in a dynamic environment where future itineraries are unknown and maintenance capacity limits the number of jets that can be maintained on a given day. Thus, there is an

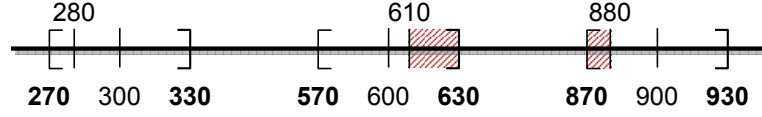
allowance on both sides of  $H$ , i.e.

- (R1) A jet should accumulate at least  $H^{min} = H - w$  hours of flying time before its next maintenance, and
- (R2) A jet can accumulate at most  $H^{max} = H + w$  hours of flying time before its next maintenance.

Over the lifetime of a jet, using only these two rules might not provide a maintenance schedule in accordance with the original intentions. If a jet is maintained always after accumulating  $H^{max}$  ( $H^{min}$ ) hours, then in the long run it is maintained less (more) frequently than originally intended. In other words, if a jet is maintained early in one interval, we would like to maintain it later in the next interval to ensure that on average it accumulates close to the target number of flying hours before being maintained. Thus, we introduce another rule to determine the timing of maintenance activities over the lifetime of a jet:

- (R3) The  $n^{th}$  maintenance can only be done when a jet accumulates flying hours in the interval  $[nH - w .. nH + w]$  (the integer interval between  $nH - w$  and  $nH + w$ ).

For example, consider the case where  $H = 300$  and  $w = 30$ . The intervals of accumulated flying hours in which a jet can be maintained are shown in Figure 2.1. The first maintenance can be done when the jet accumulates flying hours in the interval  $[270 .. 330]$ . Suppose it is maintained when it accumulates 280 hours. Considering the minimum and maximum flying hours to be accumulated before maintenance, the next maintenance can be done when the jet accumulates flying hours between  $280 + 270 = 550$  and  $280 + 330 = 610$ . However, that maintenance should also fall into the interval  $[570 .. 630]$ . Thus, the next maintenance for this jet can only be



**Figure 2.1:** Allowable maintenance intervals for a jet

done when it accumulates flying hours in the interval  $[570 .. 610]$  and the shaded area within the second maintenance interval as shown in Figure 2.1 becomes unavailable.

The aim of tactical capacity planning is to ensure that maintenance is performed according to the rules (R1) - (R3) while jets accumulate certain flying hours on each day. At the tactical level planning, we assume that jets accumulate an average number of flying hours denoted by  $f$  on each day of the planning horizon. At the operational level, jets can be assigned itineraries with different flying hours on each day so that the average number of hours they fly per day is close to  $f$ , the historical average.

## ***2.2 Tactical Planning for Maintenance Capacity in the Growth Phase***

Tactical planning for scheduled maintenance in the growth phase involves determining the daily capacities at the maintenance facility for a given planning horizon during which jets are introduced gradually into the fleet at specified points in time. The workload at the maintenance facility increases over time as new jets are introduced into the fleet. Since capacity installments are costly, any additional capacity installed to cope with increasing workload is never discarded and thus, capacity is monotone non-decreasing over time. Each unit of maintenance capacity also has a corresponding cost associated with the maintenance personnel. Thus, the objective of tactical maintenance capacity planning in the growth phase is to achieve the lowest possible total capacity over the planning horizon while scheduling maintenance in a timely

fashion.

Before presenting the integer programming formulation for the General Tactical Capacity Planning Problem (GTCP), we introduce some notation. Let  $\mathcal{T}$  be the planning horizon and  $\mathcal{J}$  be the set of all jets introduced over the planning horizon. As mentioned before, jets are introduced gradually over time. Let  $d_j$  denote the day jet  $j$  is introduced into the fleet. Note that when jets are introduced into the fleet, they start with 0 accumulated flying hours. Let  $\left\lceil \frac{H^{min}}{f} \right\rceil = n^{min}$  and  $\left\lfloor \frac{H^{max}}{f} \right\rfloor = n^{max}$ . Then, the set of days jet  $j$  can be maintained for the first time, denoted by  $\mathcal{M}_j^F$ , is  $\{d_j + n^{min} - 1, \dots, d_j + n^{max} - 1\}$ . Let  $\mathcal{M}_{jt}^N$  denote the set of days jet  $j$  can be maintained next given that it is maintained on day  $t$ , then  $\mathcal{M}_{jt}^N = \{t + n^{min}, \dots, t + n^{max}\} \cap \{d_j - 1 + \left\lceil \frac{n'H - w}{f} \right\rceil, \dots, d_j - 1 + \left\lfloor \frac{n'H + w}{f} \right\rfloor\}$  where  $n' = \min\{n \in \mathbb{Z}^+ : nH > (t - d_j + 1)f + H^{min}\}$ . As can be seen,  $\mathcal{M}_{jt}^N$  is the intersection of two intervals where the first interval is obtained by considering maintenance rules (R1) and (R2), and the second interval is obtained by considering maintenance rule (R3).

There are two types of decision variables in the formulation. The binary variable  $x_{st}^j$  equals 1 if jet  $j$  is maintained on day  $s$  and next on day  $t$ , and 0 otherwise. The integer variable  $cap_t$  represents the capacity on day  $t$ . The integer programming formulation for GTCP is:

$$(GTC P) : \min z = \sum_{t \in \mathcal{T}} cap_t \quad (2.1)$$

s.t.

$$\sum_{t \in \mathcal{M}_j^F} x_{d_j-1,t}^j = 1 \quad \forall j \in \mathcal{J} \quad (2.2)$$

$$\sum_{s \in \mathcal{T} : t \in \mathcal{M}_{js}^N} x_{st}^j = \sum_{s \in \mathcal{M}_{jt}^N} x_{ts}^j \quad \forall j \in \mathcal{J}, \forall t \in \mathcal{T} \quad (2.3)$$

$$\sum_{j \in \mathcal{J}} \sum_{s \in \mathcal{T} : t \in \mathcal{M}_{js}^N} x_{st}^j \leq cap_t \quad \forall t \in \mathcal{T} \quad (2.4)$$

$$cap_t \leq cap_{t+1} \quad \forall t \in \mathcal{T} \quad (2.5)$$

$$x_{st}^j \in \{0, 1\} \quad \forall j \in \mathcal{J}, \forall s \in \mathcal{T}, \forall t \in \mathcal{M}_{js}^N \quad (2.6)$$

$$cap_t \geq 0, \text{ integer} \quad \forall t \in \mathcal{T}. \quad (2.7)$$

The objective is to minimize the total capacity over the planning horizon. Constraints (2.2) ensure that each jet is maintained once in its first maintenance interval. The periodic maintenance constraints are represented by constraints (2.3). Constraints (2.4) ensure that the number of jets maintained on each day is less than or equal to the capacity on that day. Finally, (2.5) are the monotonicity constraints on the capacity.

### 2.2.1 History-Independent Tactical Capacity Planning Problem

GTCP considers maintenance rules (R1) - (R3). To analyze the increase in the capacity due to (R3), which is not commonly used in many applications, we consider a simpler model that is concerned with only (R1) and (R2). We call this model History-Independent Tactical Capacity Planning (HTCP).

Let  $\mathcal{M}_t^N$  denote the set of days on which the next maintenance for any jet can be done given that it is maintained on day  $t$ .  $\mathcal{M}_t^N = \{t + n^{min}, \dots, t + n^{max}\}$  since the timing between successive maintenance activities is determined only by  $H^{min}$  and



$H^{max}$ . Let  $\mathcal{J}_t$  denote the set of jets introduced on day  $t$ . Variable  $z_{st}$  represents the number of jets that are maintained on day  $s$  and next on day  $t$ . HTCP is:

$$(HTCP) : \min z = \sum_{t \in \mathcal{T}} cap_t \quad (2.8)$$

*s.t.*

$$\sum_{s \in \mathcal{T} : t \in \mathcal{M}_s^N} z_{st} + |\mathcal{J}_{t+1}| = \sum_{s \in \mathcal{M}_t^N} z_{ts} \quad \forall t \in \mathcal{T} \cup \{0\} \quad (2.9)$$

$$\sum_{s \in \mathcal{T} : t \in \mathcal{M}_s^N} z_{st} \leq cap_t \quad \forall t \in \mathcal{T} \quad (2.10)$$

$$cap_t \leq cap_{t+1} \quad \forall t \in \mathcal{T} \quad (2.11)$$

$$z_{st} \geq 0, \text{ integer} \quad \forall s \in \mathcal{T}, \forall t \in \mathcal{M}_s^N \quad (2.12)$$

$$cap_t \geq 0, \text{ integer} \quad \forall t \in \mathcal{T}. \quad (2.13)$$

The periodic maintenance constraints are represented by constraints (2.9) and the maintenance capacity constraints are represented by constraints (2.10). All other constraints and the objective function are the same as the ones in GTCP. Note that this formulation considers the jets maintained on a given day in an aggregate fashion and does not model maintenance decisions for individual jets.

It is easy to see that a feasible solution for GTCP can be aggregated into a feasible solution for HTCP by summing the values of the maintenance decision variables  $x_{st}^j$  over the jets and keeping the values of the capacity decision variables the same. Furthermore, note that HTCP is an integer flow formulation with side constraints where nodes correspond to days in the planning horizon and the variable  $z_{st}$  corresponds to the flow on arc  $(s, t)$ . Thus, any feasible integer flow can be decomposed into paths corresponding to the flow of individual jets and these paths represent the maintenance schedule of the jets over the planning horizon. The decomposed solution over the paths together with the corresponding values of the capacity decision variables

obtained while solving HTCP constitute a feasible solution for GTCP.

**Proposition 1.** *HTCP is equivalent to GTCP when rule (R3) is omitted.*

### 2.2.2 Solution Approach

An integral part of the solution approach is strengthening the formulations by including additional valid inequalities. The set of jets that have been introduced earlier than day  $t$  are referred to as *active jets* on that day and the set of active jets on day  $t$  is denoted by  $\mathcal{J}_t^A$ , i.e  $\mathcal{J}_t^A = \{j : d_j \leq t\}$ . Let  $\alpha_t = \left\lfloor \frac{|\mathcal{J}_t^A|}{n^{max}} \right\rfloor$ , and  $\beta_t = |\mathcal{J}_t^A| \bmod n^{max}$ . Note that  $n^{max}$  is the largest number of days a jet can go without maintenance for both GTCP and HTCP.

**Proposition 2.** *For  $t \in \mathcal{T}$ ,*

$$\sum_{t'=t+n^{max}-\beta_t}^{t+n^{max}-1} cap_{t'} \geq \beta_t(\alpha_t + 1) \quad (2.14)$$

*is a valid inequality for GTCP and HTCP. That is, for each day  $t \in \mathcal{T}$ , the total capacity on the last  $\beta_t$  days of the interval  $\{t, \dots, t + n^{max} - 1\}$  should be at least  $\beta_t(\alpha_t + 1)$ .*

*Proof.* Suppose not. Then,  $cap_{t+n^{max}-\beta_t} \leq \alpha_t$ . Otherwise, we would have  $cap_{t'} \geq \alpha_t + 1, \forall t' \geq t + n^{max} - \beta_t$  and (2.14) would hold.

Since  $cap_{t+n^{max}-\beta_t} \leq \alpha_t$ ,  $cap_{t'} \leq \alpha_t \quad \forall t' \leq t + n^{max} - \beta_t$  from the monotone non-decreasing property. Thus,

$$\sum_{t'=t}^{t+n^{max}-\beta_t-1} cap_{t'} \leq \alpha_t(n^{max} - \beta_t)$$

which implies

$$\sum_{t'=t}^{t+n^{max}-1} cap_{t'} < \beta_t(\alpha_t + 1) + \alpha_t(n^{max} - \beta_t) = \alpha_t n^{max} + \beta_t = |\mathcal{J}_t^A|.$$

Since the total capacity in the next  $n^{max}$  days starting from day  $t$  is less than the number of active jets on day  $t$ , at least one of these jets would not be able to be maintained without exceeding its maximum interval. Thus, we arrive at a contradiction and conclude that (2.14) is a valid inequality.  $\square$

From now on we assume that GTCP and HTCP refer to the formulations in which inequalities (2.14) are added. HTCP is solved routinely by CPLEX. The relatively smaller size of this problem and the strengthening in the LP relaxation provided by the valid inequalities make it possible to solve it in seconds. For GTCP, where we model each jet separately in order to consider their maintenance histories, it is generally not possible to find a good solution in a reasonable amount of time even with the added valid inequalities. Finding good solutions in a reasonable amount of time even for planning purposes of this type is necessary to conduct sensitivity analysis to see the impact of changing problem parameters. For example, we need to understand the effect of the fleet introduction schedule on the capacity to negotiate the best such schedule. Thus, we introduce an optimization-based local search algorithm to solve GTCP.

The optimization-based local search algorithm starts with an initial solution constructed trivially as follows. Suppose all maintenance activities are performed as early as possible considering maintenance rules (R1) - (R3). That is,  $x_{d_j-1,t}^j = 1$  if  $t$  is the first day in  $M_j^F$  and given that jet  $j$  is maintained on day  $s$ ,  $x_{st}^j = 1$  if  $t$  is the first day in  $M_{js}^N$ . Furthermore,  $cap_t = cap_{t-1}$  if the total number of jets maintained on day  $t$  is less than or equal to the number of jets maintained on day  $t - 1$ . Otherwise,  $cap_t$  is equal to the number of jets maintained on day  $t$ .

A neighborhood around a solution is constructed by fixing the maintenance decision variables corresponding to all but a subset of jets to their values in this solution. Let  $\bar{S} = \{\bar{x}_{st}^j, \bar{cap}_t, \forall j \in \mathcal{J}, \forall s, t \in \mathcal{T}\}$  be a feasible solution for GTCP and  $\mathcal{J}_{\bar{S}}$  be

the subset of jets for which the maintenance decisions are not fixed. The corresponding neighborhood, denoted by  $N_{\bar{\mathcal{J}}, \mathcal{J}_{\bar{\mathcal{J}}}}$ , is  $\{x_{st}^j, cap_t : x_{st}^j, cap_t \text{ satisfy the constraints of GTCP, } x_{st}^j = \bar{x}_{st}^j, \forall j \in \mathcal{J} \setminus \mathcal{J}_{\bar{\mathcal{J}}}\}$ .

$\mathcal{J}_{\bar{\mathcal{J}}}$  is chosen such that the optimization problem for the corresponding neighborhood is likely to lead to improvement and is efficiently solvable. The first type of neighborhood, referred to as the *primary neighborhood*, is obtained by choosing  $\mathcal{J}_{\bar{\mathcal{J}}}$  among the set of jets that are maintained on the days where capacity increases denoted by  $\mathcal{J}^{inc}$ . The aim of choosing  $\mathcal{J}_{\bar{\mathcal{J}}}$  this way is to delay the days where capacity increases by changing the decisions for the jets that are maintained on these days, and thus decreasing the objective function value. In order to solve the corresponding optimization problem in the neighborhood efficiently, only a specified number, denoted by  $r_1$ , of such jets are randomly chosen to be included in  $\mathcal{J}_{\bar{\mathcal{J}}}$ .

Our experiments indicated that if an improving solution cannot be obtained with the chosen primary neighborhood, exploring different neighborhoods of this type by selecting different subsets from  $\mathcal{J}^{inc}$  did not lead to an improvement for many iterations. Thus, a *secondary neighborhood*, is constructed to increase the likelihood of obtaining an improving solution in this situation. The secondary neighborhood is obtained by choosing  $\mathcal{J}_{\bar{\mathcal{J}}}$  to include jets that are not in  $\mathcal{J}^{inc}$  as well as jets that are in  $\mathcal{J}^{inc}$ . Specifically,  $r_1$  jets are chosen randomly among the ones in  $\mathcal{J}^{inc}$  and  $r_2$  jets are chosen randomly among the ones that are not in  $\mathcal{J}^{inc}$ .

The optimization-based local search algorithm starts with a chosen primary neighborhood. The neighborhoods are explored by solving the corresponding optimization problems using CPLEX with an upper bound on the solution time for each neighborhood. If at any iteration, an improving solution cannot be obtained within the chosen primary neighborhood, the local search starts exploring secondary neighborhoods until an improving solution is obtained. After obtaining an improving solution, it goes back to exploring primary neighborhoods. The search stops when a specified

time limit is reached.

### 2.2.3 Computational Results

We report the results of our solution approach for instances that represent 10 practical scenarios for the growth of the fleet over a two year planning horizon. The aim of the computational study is to analyze both the impact of the frequency in which jets are introduced into the fleet as well as the total number of jets required to meet customer demand. We consider two classes of instances: The first class assumes that at the end of the second year we have as many jets as the number of business days in the planning horizon, i.e. on average one new jet arriving per day and 480 jets at the end of the planning horizon. The second class assumes that projected customer demand warrants approximately half the number of jets over the planning horizon, i.e. on average one new jet arriving every two days and 288 jets at the end of the planning horizon. Finally, within each of these two classes we test five possibilities for the frequency in which jets arrive: every 1 – 2 days, 1 week, 2 weeks, 1 month, and 2 months. The arrival of new jets has a delayed impact on maintenance capacity as jets have to accumulate a minimum number of flying hours before maintenance. In order to account for the delay for jets introduced at the end of the second year, an additional six month period is included after two years.

Scheduled maintenance is required every 300 hours on average. An allowance of 10% is given from both sides of 300. Also, maintenance rule (R3) translates to maintenance having to fall into  $300n \pm 30$  hours intervals where  $n \in \mathbb{Z}^+$ . Finally, the average flying time  $f$  is 10 hours. The code is written in C++ and the IP models are built and solved with Concert 2.1 in ILOG CPLEX 9.1 using 2.4 GHz AMD 250 processors with 4 GB of RAM. Initial experiments indicated that the root node solved fastest when using the Barrier algorithm. We thus report results where the Barrier algorithm with crossovers is used to solve the root node and switching to default

CPLEX then onwards.

During the implementation of the optimization-based local search algorithm, we experimented with different values for the sizes of the subset of jets for which we optimize the maintenance decisions in the primary and secondary neighborhoods. The results of these experiments indicated that the following configuration gives the best quality solutions in a reasonable amount of time: primary neighborhoods including  $r_1 = 20$  jets chosen from  $J^{inc}$ , the set of jets maintained on the days where capacity increases, and the secondary neighborhoods including  $r_1 = 20$  jets chosen from  $J^{inc}$  and  $r_2 = 20$  jets chosen among the ones that are not in  $J^{inc}$ . Furthermore, the time limit for exploring a neighborhood is set to be 120 seconds.

Figure 2.2 shows the progress of the optimization-based local search over time during the first half an hour in terms of gap to the best lower bound obtained within 24 hours with CPLEX while Figure 2.3 shows the same during the time period between half an hour and two hours. The instances are named according to the number of jets they have at the end of the planning horizon and the time interval between jet arrivals. For example,  $(480J, 1M)$  represents the instance in which a total of 480 jets are introduced where the time interval between jets arrivals is 1 month.

As can be seen from Figure 2.2, the optimization-based local search algorithm finds a solution with a gap under or very close to 5% within 10 minutes for half of the instances. Furthermore, all instances are solved to less than 5% of optimality within half an hour. All instances are solved to around 1% of optimality and the progress rate becomes significantly small by the end of two hours as can be seen from Figure 2.3.

It is interesting to note that the optimization-based local search algorithm starts with a solution that has a very small gap, which is around 10%, only for the instance in which a total of 480 jets are introduced on average every day. On examining the total number of maintenances in the initial solution as a percentage of total capacity

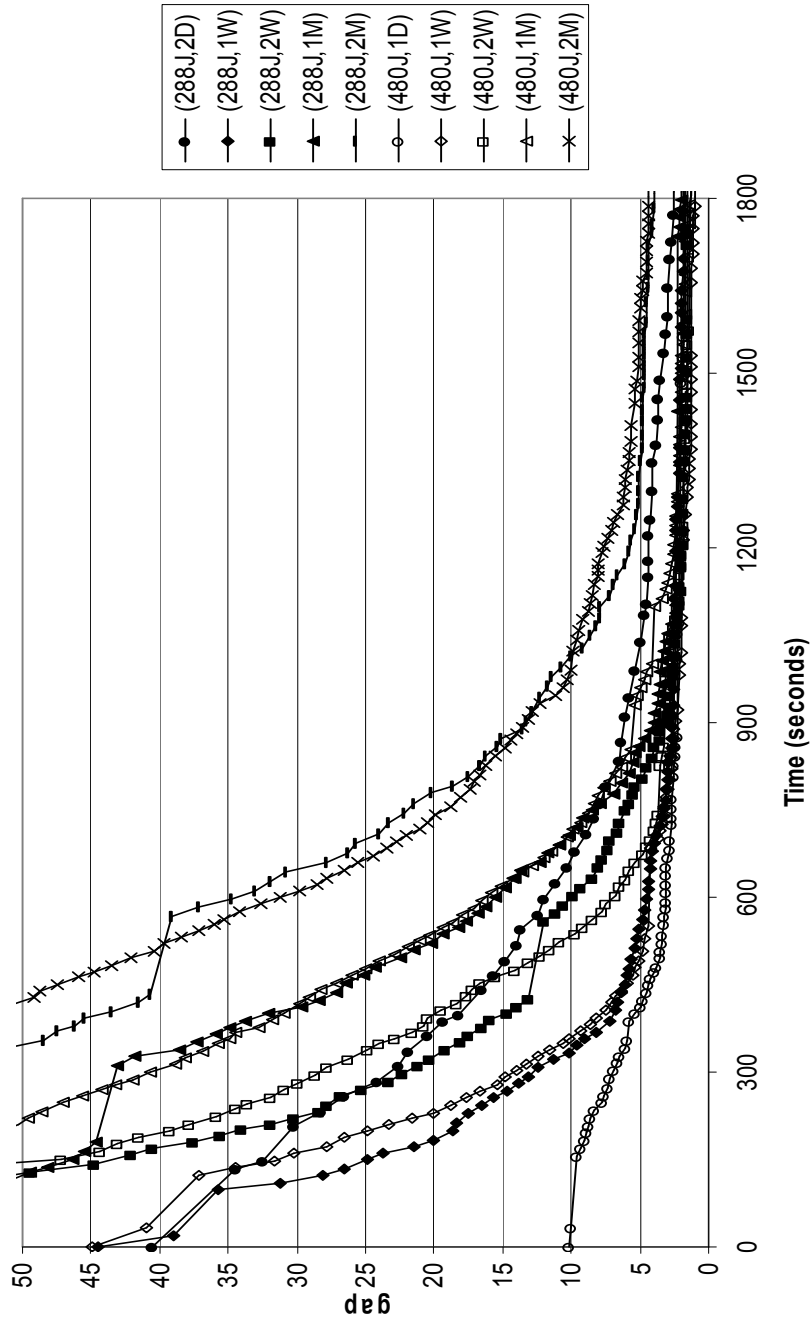


Figure 2.2: Optimization-based local search progress within half an hour

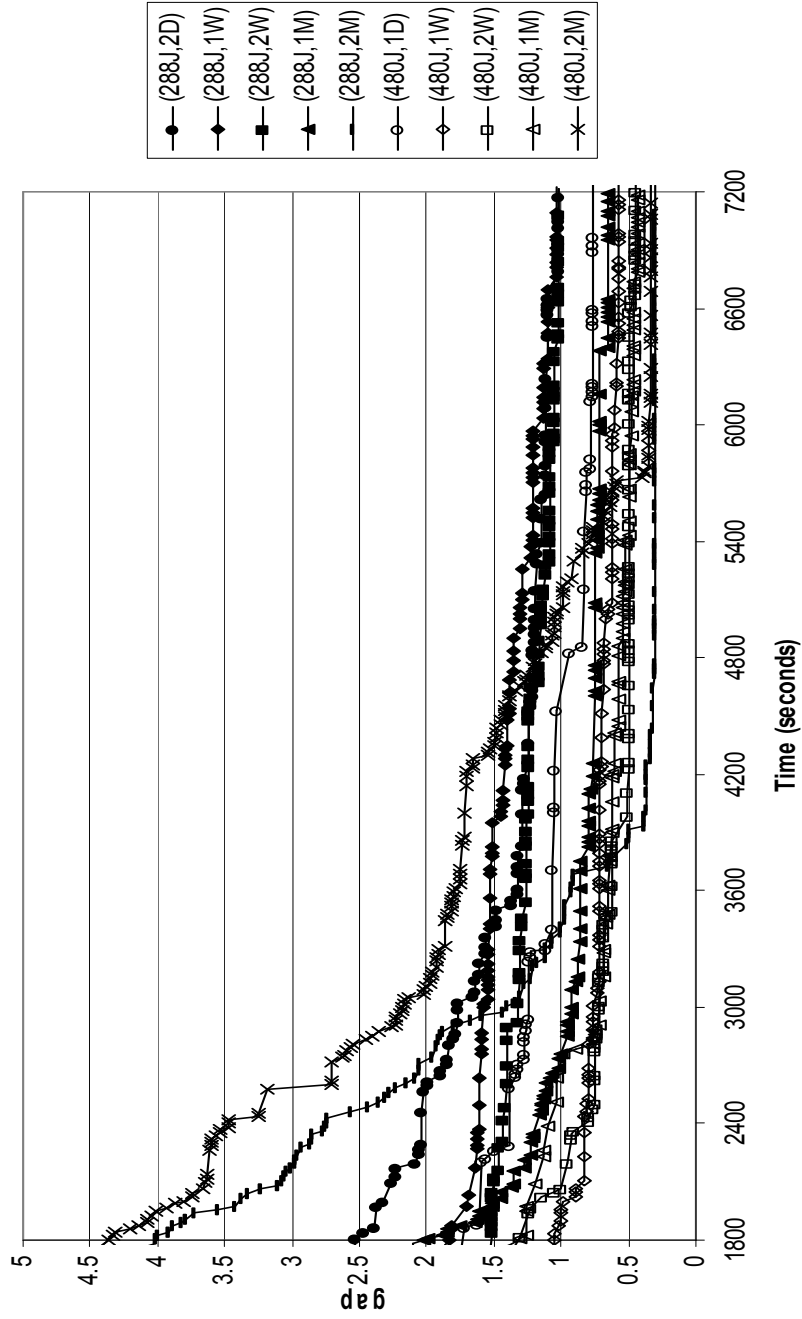


Figure 2.3: Optimization-based local search progress between half an hour and two hours



allocated, we note that 89% of the allocated capacity is already used which indicates that there is little room for improvement. For all other instances, only around 40–50% of the total allocated capacity is used in the initial solution. This implies that in these instances it is possible to decrease the total capacity by delaying the capacity increases and still comply with the maintenance rules.

We compare the results of our optimization-based local search algorithm to the results obtained by solving GTCP using CPLEX in Table 2.1. The first column corresponds to the instance names. The next two columns represent the best upper bound, denoted by ***BEST UB***, and the best lower bound, denoted by ***BEST LB***, obtained within 24 hours with CPLEX respectively. These runs are used as a benchmark to judge the quality of solutions obtained in shorter time. In the rest of the table, we present the objective values (***O.V.***) and the gap of these objective values to ***BEST LB (gap)*** within half an hour, one hour and two hours time limits for CPLEX and the optimization-based local search algorithm, respectively. Finally, the last column in Table 2.1 (***Optimal Without (R3)***) represents the optimal total capacity that would be allocated if maintenance rule (R3) was not considered.

As can be seen from Table 2.1, CPLEX found a feasible solution for none of the instances within half an hour whereas feasible solutions with on average 2% optimality gap are obtained with the optimization-based local search. Furthermore, less than half of the instances are solved to on average 4.68% and 2.22% optimality with CPLEX within one hour and two hours, respectively while all instances are solved to on average 1% and 0.65% optimality with optimization-based local search within one hour and two hours, respectively. These results clearly prove the benefits that can be achieved with optimization-based local search, especially for the larger 480 jets instances as CPLEX finds a feasible solution for none of these instances within two hours. In addition, during the 24 hour CPLEX runs, although most 288 jets instances end up with a better objective value than the objective value of the solutions found with

**Table 2.1:** Summary results for the tactical capacity planning problem

Instance	CPLEX								Optimization-based Local Search								Optimal Without (R3)
	24 hours		30 min.		1 hour		2 hours		30 min.		1 hour		2 hours				
	Best UB	Best LB	O.V.	gap	O.V.	gap	O.V.	gap	O.V.	gap	O.V.	gap	O.V.	gap			
(288J, 2D)	5075	5057.8	-	-	5394	6.23	5075	0.34	5189	2.52	5126	1.33	5110	1.02	4788		
(288J, 1W)	5138	5076.8	-	-	5472	7.22	5402	6.02	5173	1.86	5156	1.53	5130	1.04	4810		
(288J, 2W)	5273	5152	-	-	-	-	-	-	5232	1.51	5218	1.26	5205	1.02	4874		
(288J, 1M)	5332	5322.8	-	-	-	-	5463	2.57	5433	1.99	5369	0.86	5358	0.66	5016		
(288J, 2M)	6173	6171.8	-	-	6214	0.68	6173	0.02	6430	4.01	6230	0.93	6191	0.31	5531		
(480J, 1D)	-	8335.4	-	-	-	-	-	-	8486	1.78	8425	1.07	8398	0.75	7710		
(480J, 1W)	-	8394.2	-	-	-	-	-	-	8483	1.05	8455	0.72	8442	0.57	7799		
(480J, 2W)	-	8547	-	-	-	-	-	-	8668	1.4	8601	0.63	8585	0.44	7947		
(480J, 1M)	-	8746.2	-	-	-	-	-	-	8865	1.34	8804	0.66	8784	0.43	8224		
(480J, 2M)	10106	10051	-	-	-	-	-	-	10513	4.39	10235	1.8	10084	0.33	9026		

the optimization-based local search algorithm in two hours, CPLEX found feasible solutions for almost none of the 480 jets instances.

For a fixed level of jets at the end of the planning horizon, it is expected that the total capacity would be lower for the instances in which a smaller number of jets are introduced more frequently since there would be more opportunities for spreading the maintenance over time. The results in Table 2.1 substantiate this expectation. However, the rate of change in total capacity diminishes as the introductions become more frequent since the length of the extra interval to spread the maintenance gets smaller.

Finally, if we compare the solution values that are obtained with the optimization-based local search algorithm within two hours and the optimal total capacity that would be achieved if maintenance rule (R3) was not considered, we see that the total capacity increases by around 7% which is a significant investment to accommodate this rule. This additional investment can be considered as the hedge against potential costs of increased frequency of maintenance or breakdowns if this rule is not followed.

### ***2.3 Tactical Planning for Steady State Maintenance Capacity***

Tactical planning for steady state maintenance capacity involves determining the daily capacities for an infinite horizon during which the fleet size is constant. In steady state, the capacity at the maintenance facility reaches a constant as the workload does not increase over time. Thus, the objective of tactical planning in steady state is to determine the minimum capacity that allows scheduling maintenance in a timely fashion.

Consider the infinite horizon problem where a set of machines has to be maintained periodically and the time interval until the next maintenance for each machine is constant. Determining the capacity needed for a feasible maintenance schedule for these machines is shown to be NP-hard in Mok et al. [27]. In steady state, our tactical

maintenance capacity planning problem generalizes this NP-hard problem as each jet can be maintained in an interval of days rather than on a single day after its previous maintenance and thus, is NP-hard. In this section, we analyze a special case of tactical capacity planning for scheduled maintenance of PSOD air transportation where the optimal and the long run capacities can be determined with a pseudo-polynomial time algorithm.

The special case is obtained as follows. Jets are introduced into the system such that the same number of jets arrive on each day of the set  $\{t^s, \dots, t^e\}$  and all jets arrive before any jet needs maintenance. As the number of jets reaches its maximum before maintenance needs to be scheduled and stays constant from then on, this is a steady state situation. Furthermore, maintenance rule (R3) is not considered. Thus, the timing until the next maintenance is determined only by the minimum and the maximum flying hours to be accumulated, i.e. by maintenance rules (R1) and (R2).

We define the  $k^{th}$  maintenance interval, denoted by  $\mathcal{I}_k^M$ , to be the set of all days on which any jet can be maintained for the  $k^{th}$  time. Then,  $\mathcal{I}_k^M = \{t^s + kn^{min} - 1, \dots, t^e + kn^{max} - 1\}$ . Note that depending on when a jet was first introduced or when it was previously maintained, only some of the days in  $\mathcal{I}_k^M$  are possible for that particular jet to be maintained. For completeness, let  $\mathcal{I}_0^M = \{t^s, \dots, t^e\}$ .

Consider an example where jets are introduced within the interval  $\{2, \dots, 6\}$ , and we have  $n^{min} = 7$  and  $n^{max} = 9$ . The first maintenance interval,  $\mathcal{I}_1^M$ , can be given as  $\{8, \dots, 14\}$  where day 8 (day 14) is the earliest (the latest) day any jet can be maintained for the first time. However, a jet that is introduced on day 4 can be maintained for the first time only in the interval  $\{10, 11, 12\}$ .

**Assumption 3.**  $n^{min} < n^{max}$ . If  $n^{min} = n^{max}$ , the problem would be trivial since all the maintenance days would be predetermined for the jets.

### 2.3.1 Properties of the maintenance intervals

In this section, we discuss some properties of the maintenance intervals that will provide the basis for developing an algorithm to generate a feasible maintenance schedule. Let  $l_k$  denote the length of  $\mathcal{I}_k^M$ .

**Observation 4.** *The lengths of the maintenance intervals are monotone increasing since  $\forall k \in \mathbb{Z}^+$ ,  $l_k - l_{k-1} = n^{\max} - n^{\min} > 0$ .*

**Claim 5.**  *$\exists k \in \mathbb{Z}^+$  such that  $\mathcal{I}_k^M$  and  $\mathcal{I}_{k+1}^M$  intersect, that is maintenance intervals intersect at some point.*

*Proof.* Suppose not. Then, we would have  $t^s + (k+1)n^{\min} - 1 > t^e + kn^{\max} - 1$ ,  $\forall k$ .

Thus,

$$\frac{k+1}{k}n^{\min} > \frac{t^e - t^s}{k} + n^{\max}, \quad \forall k.$$

$$\text{As } k \rightarrow \infty, \frac{k+1}{k} \rightarrow 1 \text{ and } \frac{t^e - t^s}{k} \rightarrow 0.$$

This implies that in order for the maintenance intervals not to intersect we should have  $n^{\min} > n^{\max}$  which contradicts Assumption 3.  $\square$

Let  $\mathcal{I}_{\bar{k}}^M$  be the first interval to intersect with its next maintenance interval, i.e.  $\bar{k} = \min\{k \in \mathbb{Z}^+ : \mathcal{I}_k^M \cap \mathcal{I}_{k+1}^M \neq \emptyset\}$ .

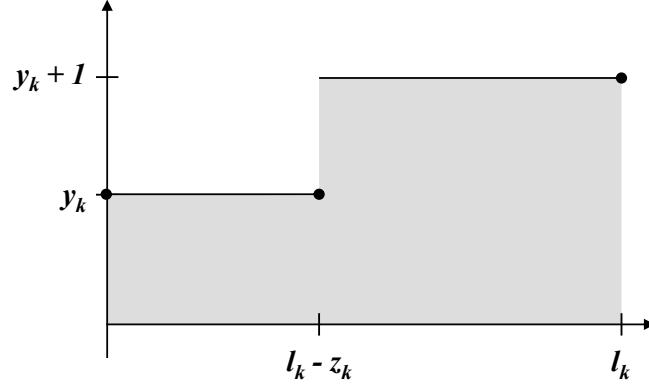
**Observation 6.**  *$\forall k > \bar{k}$ ,  $\mathcal{I}_k^M$  and  $\mathcal{I}_{k+1}^M$  intersect since  $t^s + (\bar{k} + a + 1)n^{\min} - 1 < t^e + (\bar{k} + a)n^{\max} - 1$ ,  $\forall a \in \mathbb{Z}^+$  given that  $n^{\min} < n^{\max}$ . In other words, all maintenance intervals after  $\mathcal{I}_{\bar{k}}^M$  continue to intersect with their next intervals.*

**Claim 7.**  *$\forall k \leq \bar{k}$ ,  $l_k \leq n^{\max}$ .*

*Proof.* Since  $\mathcal{I}_k^M$  and  $\mathcal{I}_{k-1}^M$  do not intersect, we have  $t^e + (k-1)n^{\max} - 1 < t^s + kn^{\min} - 1$ .

Thus,  $l_k = t^e - t^s + 1 + k(n^{\max} - n^{\min}) < kn^{\min} - (k-1)n^{\max} + 1 + kn^{\max} - kn^{\min}$  which implies  $l_k \leq n^{\max}$ .  $\square$

**Observation 8.**  *$\forall k \geq \bar{k} + 1$ , the length of the part of  $\mathcal{I}_k^M$  that does not intersect with  $\mathcal{I}_{k-1}^M$  is  $t^e + kn^{\max} - 1 - (t^e + (k-1)n^{\max}) + 1 = n^{\max}$ .*



**Figure 2.4:** Smooth Maintenance Schedule  $P_k$  for  $k \leq \bar{k}$

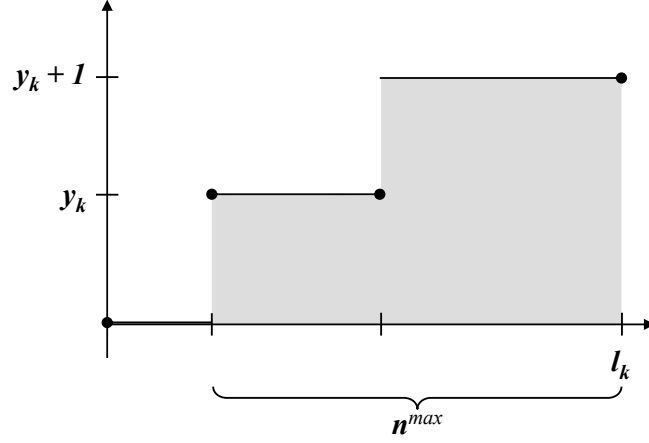
### 2.3.2 Generating a Feasible Maintenance Schedule

Our objective is to set the maintenance capacity as low as possible while scheduling maintenance according to rules (R1) and (R2). Thus, we try to distribute the number of jets maintained on each day as evenly as possible and we would like to obtain a maintenance schedule as follows:

(i) For  $k \leq \bar{k}$ , the jets are scheduled for maintenance such that the number of jets maintained on each day in  $\mathcal{I}_k^M$  is approximately the same as shown in Figure 2.4 where  $y_k = \left\lfloor \frac{|\mathcal{J}|}{l_k} \right\rfloor$  and  $z_k = |\mathcal{J}| \bmod l_k$ .

(ii) For  $k > \bar{k}$ , the jets are scheduled for maintenance within the part of the maintenance interval that does not intersect with the previous one and the number of jets maintained on each day is approximately the same as shown in Figure 2.5. From this point on,  $\mathcal{I}_k^M$  will refer to the part that does not intersect with the previous maintenance interval for  $k > \bar{k}$ . Let  $y_k = y_{n^{max}} = \left\lfloor \frac{|\mathcal{J}|}{n^{max}} \right\rfloor$  and  $z_k = z_{n^{max}} = |\mathcal{J}| \bmod n^{max}$  for  $k > \bar{k}$ .

Let the number of jets introduced on each day within the interval  $\{t^s, \dots, t^e\}$  be denoted by  $v$ . From Observation 4 and Claim 7, we have  $l_0 < l_k < l_{k+1} \leq n^{max}$  for



**Figure 2.5:** Smooth Maintenance Schedule  $P_k$  for  $k > \bar{k}$

$k \leq \bar{k}$ . Thus, we have  $v \geq y_k \geq y_{k+1} \geq y_{n^{max}}$ . Indeed,  $v > y_k$ . Suppose not, i.e  $v = y_k$ . Then, we would have  $v(t^e - t^s + 1) = v(t^e - t^s + 1 + k(n^{max} - n^{min})) + z_k$  which contradicts Assumption 3.

**Observation 9.** For  $k \leq \bar{k}$ ,  $v > y_k \geq y_{k+1} \geq y_{n^{max}}$ .

$P_0$  corresponds to the schedule in which the jets are introduced and has a smooth structure as the same number of jets are introduced on each day. Given a schedule  $P_k$  for  $k \leq \bar{k}$  as shown in Figure 2.4, Algorithm 2.1 is used to schedule the next maintenance of the jets. The algorithm starts from the first day of the current maintenance interval  $\mathcal{I}_k^M$  and schedules the next maintenance of the jets on the earliest possible day of  $\mathcal{I}_{k+1}^M$  until the number of jets scheduled for maintenance on such earliest day does not exceed the target number to achieve a schedule as shown in Figure 2.4 (Figure 2.5 if  $k = \bar{k}$ ). For each day  $t \in \mathcal{I}_k^M$ ,  $\mathcal{J}_t^M$  denotes the set of jets maintained on that day and  $n_t = |\mathcal{J}_t^M|$  (If  $k = 0$ , then  $\mathcal{J}_t^M = \mathcal{J}_t$ ). For each day  $t' \in \mathcal{I}_{k+1}^M$ ,  $n_{t'}$  denotes the number of jets needed to be assigned for maintenance on that day to obtain a smooth maintenance schedule and  $\mathcal{J}_{t'}^M$  denotes the set of jets that will be assigned for maintenance on day  $t'$ .

---

**Algorithm 2.1** FWDALLOCATION

---

**Require:**  $P_k$ .**Ensure:**  $P_{k+1}$ .

```
1:  $\mathcal{J}_{t'}^M \leftarrow \{\emptyset\}, \forall t' \in \mathcal{I}_{k+1}^M$ 
2:  $t' \leftarrow$  the first day of  $\mathcal{I}_{k+1}^M$ 
3: for  $t = t^s + kn^{\min} - 1$  to  $t^e + kn^{\max} - 1$  do
4:   for  $j$  in  $\mathcal{J}_t^M$  do
5:     if  $|\mathcal{J}_{t'}^M| = n_{t'}$  then
6:        $t' \leftarrow t' + 1$ 
7:     end if
8:      $\mathcal{J}_{t'}^M \leftarrow \mathcal{J}_{t'}^M \cup \{j\}$ 
9:   end for
10: end for
```

---

Algorithm 2.2 summarizes the procedure for obtaining the target maintenance schedule. In order to schedule the maintenance of jets in  $\mathcal{I}_k^M$  for  $k \leq \bar{k} + 1$ , we use Algorithm 2.1 starting from  $\mathcal{I}_0^M$ . For  $k > \bar{k} + 1$ , jets are maintained when they accumulate exactly  $H^{\max}$  flying hours after their previous maintenance and thus, a jet is maintained  $n^{\max}$  days after its previous maintenance. That is, the maintenance schedule obtained for  $\mathcal{I}_{\bar{k}+1}^M$  repeats itself indefinitely.

---

**Algorithm 2.2** SCHEDULE

---

```
1: for  $k = 0$  to  $\bar{k}$  do
2:   Use  $FwdAllocation(P_k)$  to obtain  $P_{k+1}$ .
3: end for
4: Use  $P_{k+1}, \forall k > \bar{k} + 1$ .
```

---

Note that if  $\mathcal{I}_k^M$  does not intersect with  $\mathcal{I}_{k+1}^M$ , we have  $t^e + kn^{\max} < t^s + (k+1)n^{\min}$ . For such an interval the following holds:  $t^e - t^s + k(n^{\max} - n^{\min}) < n^{\min}$ . Since all jets arrive before the first maintenance interval,  $t^e < t^s + n^{\min} - 1$ . Thus, we conclude that if  $k < \frac{n^{\min} - (t^e - t^s)}{(n^{\max} - n^{\min})}$ , then  $\mathcal{I}_k^M$  and  $\mathcal{I}_{k+1}^M$  do not intersect. For all  $k$  greater than  $\frac{n^{\min} - (t^e - t^s)}{(n^{\max} - n^{\min})}$ ,  $\mathcal{I}_k^M$  and  $\mathcal{I}_{k+1}^M$  intersect. This implies that the number of iterations in Algorithm 2.2 is bounded above by  $\frac{n^{\min} - (t^e - t^s)}{(n^{\max} - n^{\min})}$  which depends on input data. Furthermore, at each iteration the number of operations made to schedule the next maintenance of jets by using either Algorithm 2.1 or Algorithm 2.3 is  $O(|\mathcal{J}|)$ .



Thus, Algorithm 2.2 is a pseudo-polynomial time algorithm.

**Proposition 10.** *Algorithm 2.2 gives a feasible maintenance schedule.*

*Proof.* We need to show that no jet is scheduled for maintenance less than  $n^{\min}$  days and more than  $n^{\max}$  days after its previous maintenance or arrival,  $\forall k \leq \bar{k} + 1$ .

Consider jet  $j \in \mathcal{J}_t^M$  where  $t$  is the  $u^{\text{th}}$  day in  $\mathcal{I}_k^M$ . In order to show that jet  $j$  is not scheduled for maintenance less than  $n^{\min}$  days after its maintenance in  $\mathcal{I}_k^M$ , we consider Algorithm 2.1. Suppose jet  $j$  is scheduled for maintenance in  $\mathcal{I}_{k+1}^M$  on a day less than  $n^{\min}$  days after its maintenance in  $\mathcal{I}_k^M$ . Thus, jet  $j$  is scheduled for maintenance in the first  $u - 1$  days of  $\mathcal{I}_{k+1}^M$ . In order for this to happen, the total number of jets scheduled for maintenance on the first  $u - 1$  days of  $\mathcal{I}_k^M$  should have been less than the total number of jets needed to be assigned for maintenance on the first  $u - 1$  days of  $\mathcal{I}_{k+1}^M$ . Thus, we would have

$$\sum_{o=1}^{u-1} n_o < \sum_{o'=1}^{u-1} n_{o'}. \quad (2.15)$$

From Observation 9, we know that  $v > y_k \geq y_{k+1} \geq y_{n^{\max}}$ .

For  $k = 0$ ,  $\sum_{o'=1}^{u-1} n_{o'} < (u - 1)(y_1 + 1) \leq (u - 1)v = \sum_{o=1}^{u-1} n_o$  which contradicts (2.15). Thus, we conclude that jet  $j$  will not be scheduled for maintenance in  $\mathcal{I}_1^M$  less than  $n^{\min}$  days after its arrival.

For  $0 < k \leq \bar{k}$ , we can have two cases:

Case 1:  $y_k \geq y_{k+1} + 1$ .

$\Rightarrow \sum_{o'=1}^{u-1} n_{o'} < (u - 1)(y_{k+1} + 1) \leq (u - 1)y_k \leq \sum_{o=1}^{u-1} n_o$  which contradicts (2.15).

Case 2:  $y_k = y_{k+1}$ .

Thus,  $z_k = |\mathcal{J}| - y_k l_k$  and  $z_{k+1} = |\mathcal{J}| - y_k(l_k + (n^{\max} - n^{\min}))$  where  $n^{\max} - n^{\min} > 0$ .

$\Rightarrow z_{k+1} < z_k$ . As a result,  $\sum_{o'=1}^{u-1} n_{o'} < \sum_{o=1}^{u-1} n_o$  which contradicts (2.15).

We proved that no jet is scheduled for maintenance less than  $n^{\min}$  days after its previous maintenance or arrival. Now, in order to show that no jet is scheduled

for maintenance more than  $n^{max}$  days after its previous maintenance or arrival we introduce Algorithm 2.3.

---

**Algorithm 2.3** BWDALLOCATION

---

**Require:**  $P_k$ .

**Ensure:**  $P_{k+1}$ .

```

1:  $\mathcal{J}_{t'}^M \leftarrow \{\emptyset\}, \forall t' \in \mathcal{I}_{k+1}^M$ 
2:  $t' \leftarrow$  the last day of  $\mathcal{I}_{k+1}^M$ 
3: for  $t = t^e + kn^{max} - 1$  to  $t^s + kn^{min} - 1$  do
4:   for  $j$  in  $\mathcal{J}_t^M$  do
5:     if  $|\mathcal{J}_{t'}^M| = n_{t'}$  then
6:        $t' \leftarrow t' - 1$ 
7:     end if
8:      $\mathcal{J}_{t'}^M \leftarrow \mathcal{J}_{t'}^M \cup \{j\}$ 
9:   end for
10: end for

```

---

Algorithm 2.3 starts from the last day of the current maintenance interval  $\mathcal{I}_k^M$  and schedules the next maintenance of the jets on the latest possible day of  $\mathcal{I}_{k+1}^M$  until the number of jets scheduled for maintenance on such latest day does not exceed the target number to achieve a schedule as shown in Figure 2.4 (Figure 2.5 if  $k = \bar{k}$ ). Both Algorithm 2.1 and Algorithm 2.3 generate the same maintenance schedule for  $\mathcal{I}_{k+1}^M$  if the jets are scheduled in the increasing order of their indices in Algorithm 2.1 and in the decreasing order of their indices in Algorithm 2.3 (assuming that we index the jets in increasing order starting from the ones scheduled on the first day of  $\mathcal{I}_k^M$ ).

In order to show that jet  $j$  is not scheduled for maintenance more than  $n^{max}$  days after its maintenance in  $\mathcal{I}_k^M$ , we consider Algorithm 2.3. Suppose jet  $j$  is scheduled for maintenance in  $\mathcal{I}_{k+1}^M$  on a day more than  $n^{max}$  days after its maintenance in  $\mathcal{I}_k^M$ . Thus, jet  $j$  is scheduled for maintenance after the first  $u + (n^{max} - n^{min})$  days of  $\mathcal{I}_{k+1}^M$ . The length of  $\mathcal{I}_{k+1}^M$  is  $l_k + (n^{max} - n^{min})$ . This means that jet  $j$  is scheduled for maintenance in the last  $l_k + (n^{max} - n^{min}) - (u + (n^{max} - n^{min})) = l_k - u$  days of  $\mathcal{I}_{k+1}^M$ . In order for this to happen, the total number of jets scheduled for maintenance on the last  $l_k - u$  days of  $\mathcal{I}_k^M$  should have been less than the total number of jets needed

to be scheduled for maintenance on the last  $l_k - u$  days of  $\mathcal{I}_{k+1}^M$ . Thus, we should have

$$\sum_{o=u+1}^{l_k} n_o < \sum_{o'=u+n^{max}-n^{min}+1}^{l_k+n^{max}-n^{min}} n_{o'} \quad (2.16)$$

For  $k = 0$ ,  $\sum_{o'=u+n^{max}-n^{min}+1}^{l_k+n^{max}-n^{min}} n_{o'} \leq (l_k - u)(y_1 + 1) \leq (l_k - u)v = \sum_{o=u}^{l_k} n_o$  which contradicts (2.16). Thus, we conclude that jet  $j$  will not be scheduled for maintenance in  $\mathcal{I}_1^M$  more than  $n^{max}$  days after its arrival.

For  $0 < k \leq \bar{k}$ , we can have two cases:

Case 1:  $y_k \geq y_{k+1} + 1$ .

$\Rightarrow \sum_{o'=u+n^{max}-n^{min}+1}^{l_k+n^{max}-n^{min}} n_{o'} \leq (u - 1)(y_{k+1} + 1) \leq (u - 1)y_k \leq \sum_{o=1}^{u-1} n_o$  which contradicts (2.16).

Case 2:  $y_k = y_{k+1}$ .

Since  $z_{k+1} < z_k$  as shown above,  $\sum_{o'=u+n^{max}-n^{min}+1}^{l_k+n^{max}-n^{min}} n_{o'} < \sum_{o=1}^{u-1} n_o$  which again contradicts (2.16).

We showed that no jet is scheduled for maintenance more than  $n^{max}$  days after its previous maintenance or arrival which completes the proof.  $\square$

### 2.3.3 Determining optimal and long run capacities

Since all jets have to be maintained at least once in each maintenance interval, we have to pack them to an interval of length  $l_k$ ,  $\forall k \in \mathbb{Z}^+$ . In order to achieve that, we need a capacity of at least  $\left\lceil \frac{|\mathcal{J}|}{l_k} \right\rceil$  and since  $l_1$  is the smallest as shown in Observation 4, the tightest such lower bound is  $\left\lceil \frac{|\mathcal{J}|}{l_1} \right\rceil$ . Furthermore, all jets have to be maintained in any interval with length  $n^{max}$  since  $n^{max}$  is the maximum number of days until the next maintenance. Thus,  $\left\lceil \frac{|\mathcal{J}|}{n^{max}} \right\rceil$  is a lower bound on the optimal capacity as well. However, since  $l_1 < n^{max}$  from Claim 7,  $\left\lceil \frac{|\mathcal{J}|}{l_1} \right\rceil$  is a tighter lower bound.

**Observation 11.**  $\left\lceil \frac{|\mathcal{J}|}{l_1} \right\rceil$  is a lower bound on the optimal capacity denoted by  $c^*$ .

$\left\lceil \frac{|\mathcal{J}|}{l_1} \right\rceil$  is indeed the optimal capacity at the maintenance facility since Algorithm 2.2

generates a maintenance schedule in which the maximum number of jets maintained on a day is  $\left\lceil \frac{|\mathcal{J}|}{l_1} \right\rceil$ .

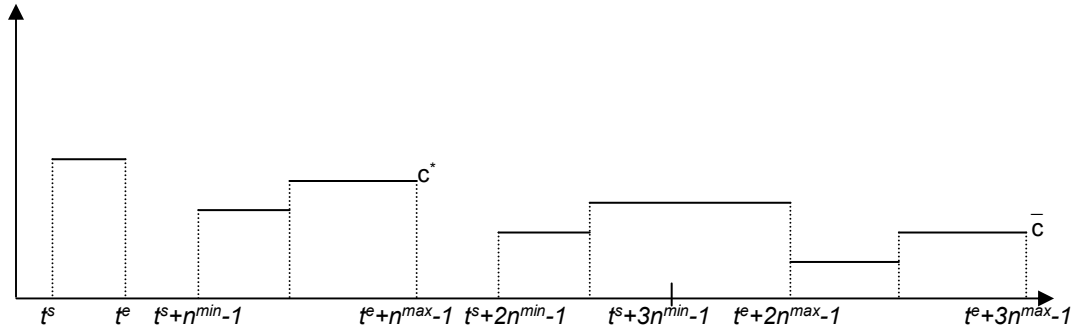
The feasible maintenance schedule obtained with Algorithm 2.2 also helps us to reach a conclusion about the long run capacity denoted by  $\bar{c}$ .

**Claim 12.** *The long run capacity needed is equal to the number of jets divided by the maximum time interval between consecutive maintenance activities rounded up; in other words  $\bar{c} = \left\lceil \frac{|\mathcal{J}|}{n^{max}} \right\rceil$ .*

*Proof.* Since the largest possible interval length to distribute the jets for maintenance is  $n^{max}$ ,  $\bar{c}$  cannot be less than  $\left\lceil \frac{|\mathcal{J}|}{n^{max}} \right\rceil$ .

Furthermore, a feasible maintenance schedule can be obtained where  $\bar{c}$  is indeed  $\left\lceil \frac{|\mathcal{J}|}{n^{max}} \right\rceil$  using Algorithm 2.2. Thus, we can conclude that  $\bar{c}$  is equal to  $\left\lceil \frac{|\mathcal{J}|}{n^{max}} \right\rceil$ .  $\square$

Figure 2.6 shows an example where the first interval to intersect with the next one is the second maintenance interval. As can be seen from the figure, the capacity required to schedule maintenance decreases monotonically until the third interval which intersects with its previous maintenance interval. The schedule obtained for the third maintenance interval will repeat itself for all the intervals after that.



**Figure 2.6:** Example Maintenance Schedule

The following example shows that the optimal and the long run capacities might be different when maintenance rule (R3) is taken into account. Consider the case when jets arrive in the interval  $\{2, \dots, 8\}$  and  $H^{min}$ ,  $H$  and  $H^{max}$  are 18, 24 and 30, respectively where  $f = 2$ . Thus,  $n^{min} = 9$  and  $n^{max} = 15$ .  $\mathcal{I}_k^M$  is found to be  $\{10 + 12(k - 1), \dots, 22 + 12(k - 1)\}$ . The first maintenance interval to intersect with the next one is  $\mathcal{I}_1^M$  where  $\mathcal{I}_1^M = \{10, \dots, 22\}$  and  $\mathcal{I}_2^M = \{22, \dots, 34\}$ . The length of the nonintersecting part of  $\mathcal{I}_2^M$  with  $\mathcal{I}_1^M$  is 12 which is smaller than  $l_1$ . Thus, the optimal capacity,  $c^*$ , is not equal to  $\left\lceil \frac{|\mathcal{J}|}{l_1} \right\rceil$ . Furthermore, the maintenance schedule that repeats itself indefinitely distributes the number of jets over an interval of length 12 which is less than  $n^{max} = 15$ , i.e. the long run capacity,  $\bar{c}$ , is not equal to  $\left\lceil \frac{|\mathcal{J}|}{n^{max}} \right\rceil$ .

# CHAPTER III

## OPERATIONAL PLANNING FOR SCHEDULED MAINTENANCE

### *3.1 Introduction*

Operational planning for scheduled maintenance involves assigning itineraries to jets and determining the specific jets to be maintained on a daily basis subject to capacity limitations. Our main objective here is to ensure that jets accumulate flying hours as close as possible to the target  $H$  between successive maintenance activities. The possibility of choosing which itineraries to assign to jets with different accumulated flying hours gives us the opportunity to optimize our maintenance decisions. However, care must be taken to avoid situations in which the number of jets maintained on the current day is much less than the capacity whereas the number of jets in need of maintenance at some future point in time is likely to exceed the capacity. Thus, we develop a solution methodology for operational maintenance planning that takes into account the impact of current decisions on the future.

Obviously, there is a strong interaction between the operational level maintenance decisions and flight scheduling. Maintenance decisions affect flight scheduling as jets that undergo maintenance cannot be used to transport passengers. Conversely, flight scheduling impacts maintenance decisions as it may be necessary to revise preferred maintenance decisions if the set of accepted transportation requests can no longer be accommodated when the preferred maintenance decisions are implemented. In order to capture this interaction, we develop a framework in which operational maintenance planning and daily flight scheduling can get feedback from each other. The framework is tested in a simulation environment where transportation requests are

generated using an agent-based model, and flight scheduling and operational maintenance planning are performed in real-time.

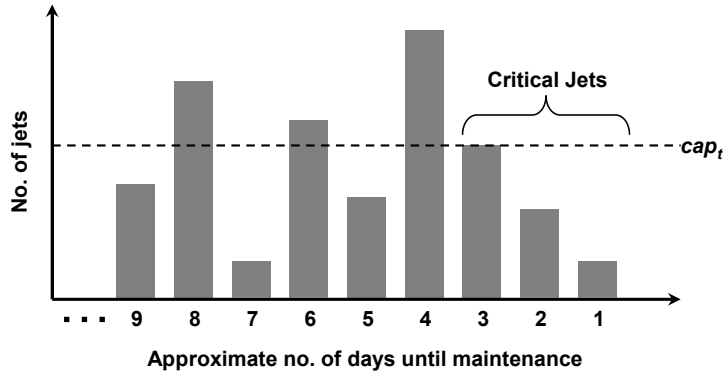
### 3.2 *Solution Approach*

We employ a look-ahead approach which involves solving a  $k$ -day problem for the current day in order to capture the effect of current maintenance decisions on the future. It might be necessary to maintain some jets earlier if the capacity would be insufficient to accommodate all the jets requiring maintenance in the future. The look-ahead approach captures that by considering information about the next  $k - 1$  days in addition to the current day. As the itineraries for a given day are constructed the night before, only the itineraries of the current day are known with certainty. For the other  $k - 1$  days, it is assumed that each jet would accumulate the average flying hours  $f$  on each day. Thus, solving a  $k$ -day problem on day  $t \in \mathcal{T}$  refers to making itinerary assignments to jets on day  $t$ , determining the jets to be maintained on each of the  $k$  days, and implementing these decisions for day  $t$ .

Maintenance rules (R1) and (R3) together determine a lower bound on the flying hours to be accumulated until the next maintenance for the jets. Thus, certain jets cannot be maintained during the  $k$  days starting from day  $t$  since their accumulated flying hours would not reach the lower bound. These jets are referred to as the *non-critical jets*. All other jets can potentially be maintained during the  $k$  days and are referred to as the *critical jets*. Exploiting the fact that non-critical jets have no impact on maintenance decisions over the  $k$  days, we decompose our problem and solve it in two phases. The first phase, which is referred to as the *maintenance decision phase*, determines the itinerary assignments to the critical jets on day  $t$  and the jets to be maintained on each of the  $k$  days. The second phase is concerned with the assignment of the remaining unassigned itineraries to the non-critical jets on day  $t$ .

Although non-critical jets play no part in maintenance decision making over the

$k$  days, assigning itineraries arbitrarily could in the long run impact the flexibility available to the maintenance decision phase to meet the target accumulated flying hours  $H$ , even worse it can lead to infeasibility with respect to maintenance capacity in the future. Consider the example given in Figure 3.1 where  $k = 3$ . Here we arrive at a situation in which although the maintenance schedule seems feasible for the next  $k$  days, the number of jets that are approximately  $k + 1$  days away from maintenance well exceeds the maintenance capacity. As a result, some jets may have to be maintained earlier or later than their intended maintenance day resulting not only in poor usage of flying hours but also possibly increasing the future workload at the maintenance facility and leading to infeasibilities. To avoid such situations, our objective in assigning itineraries to non-critical jets is to ensure that the number of jets with varying levels of accumulated flying hours is as evenly distributed as possible. We call this the *smoothing phase*.



**Figure 3.1:** Impact of large fluctuations in the number of jets with different accumulated flying hours

### 3.2.1 Maintenance Decision Phase

The Maintenance Decision Phase (MDP) is modeled as a multicommodity network flow problem on a time-expanded network denoted by  $D = (\mathcal{N}, \mathcal{A})$  where  $\mathcal{N}$  is the



set of nodes and  $\mathcal{A}$  is the set of arcs. Let  $\mathcal{T}_t$  denote the set of days MDP is solved for on day  $t$  and  $\mathcal{J}_t^C$  denote the set of critical jets on day  $t$ . Each node  $(j, t', h) \in \mathcal{N}$ , corresponds to a critical jet  $j \in \mathcal{J}_t^C$  on a given day  $t' \in \mathcal{T}_t$  with  $h$  accumulated flying hours since its last maintenance. Let  $\mathcal{I}_t$  denote the set of itineraries on day  $t$  and  $g_i$  denote the flying time of itinerary  $i$ . Suppose critical jet  $j$  starts day  $t$  with  $\bar{h}$  accumulated flying hours. The part of the network corresponding to jet  $j$  is constructed as follows. First of all, a node  $(j, t, \bar{h})$  is included in the network to represent the starting condition of jet  $j$ . Then, a node  $(j, t', 0)$  is included for each  $t' > t$  to represent the jet being maintained on day  $t' - 1$ .

On day  $t$ , for each  $i \in \mathcal{I}_t$ ,

- i) a node  $(j, t + 1, \bar{h} + g_i)$  is included in the network.
- ii) an arc is included between nodes  $(j, t, \bar{h})$  and  $(j, t + 1, \bar{h} + g_i)$  that represents jet  $j$  being assigned itinerary  $i$  on day  $t$  and not being maintained.
- iii) an arc to node  $(j, t + 1, 0)$  denoted by  $a_j^i$  is included in the network representing the jet being assigned itinerary  $i$  on day  $t$  and being maintained.

On day  $t' > t$ , for each node  $(j, t', h')$ ,

- i) a node  $(j, t' + 1, h' + f)$  is included in the network.
- ii) an arc to node  $(j, t' + 1, h' + f)$  is included representing the jet not being maintained on day  $t'$  after flying  $f$  hours.
- iii) an arc to node  $(j, t' + 1, 0)$  is included representing the jet being maintained on day  $t'$  after flying  $f$  hours.

Complying with the maintenance rules and not exceeding the capacity might not always be possible due to the uncertainty introduced into the problem by unknown itineraries. Thus, the network is constructed such that the lower bound on the flying hours to be accumulated is explicitly considered while the upper bound is relaxed. That is, we do not add arcs that violate the lower bound but we include arcs that

violate the upper bound on the flying hours to be accumulated since the last maintenance.

The objective is to maintain jets with accumulated flying hours as close to target  $H$  as possible. That is, we would like to minimize the deviation of the accumulated flying hours from the target  $H$  when jets are maintained. However, it is also important to have a schedule in which the deviations from the target is approximately uniform among the fleet. To achieve this, it is necessary to use a nonlinear penalty function. Specifically, we set the penalty of being maintained with  $H \pm q$  accumulated flying hours to be  $q^2$ .

The arcs that represent the jet being maintained are referred to as the *maintenance arcs*. The cost of arc  $a \in \mathcal{A}$  is denoted by  $c_a$  and is given by:

i)  $c_a = 0$  if  $a$  is not a maintenance arc and the jet does not accumulate flying hours larger than its upper bound by taking this arc.

ii)  $c_a = q^2$  where  $a$  is a maintenance arc on which the jet accumulates  $H \pm q$  hours before being maintained and  $H \pm q$  is less than or equal to the upper bound for the jet.

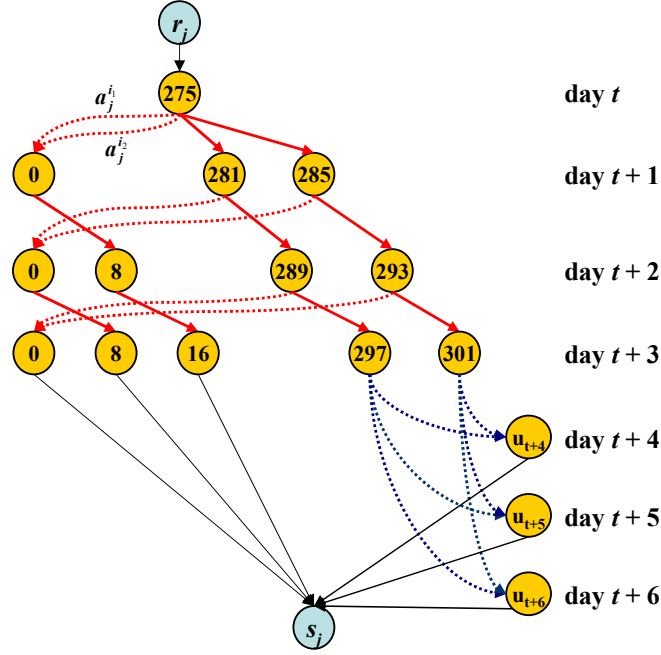
iii)  $c_a = C$  where the jet accumulates flying hours larger than the upper bound by taking arc  $a$  regardless of  $a$  being a maintenance arc or not.  $C$  is chosen to be large enough to ensure that this arc is never used by the jet instead of a feasible alternative.

Note that the costs considered so far are incurred when a jet is maintained or exceeds the upper bound on the accumulated flying hours. However, it is necessary to include a cost component to measure the effect of not maintaining a jet within  $\mathcal{T}_t$  as it will have to be maintained at some point in the near future. To this end, we consider an additional time period following  $\mathcal{T}_t$ , denoted by  $\overline{\mathcal{T}}$ , that is long enough to ensure that all critical jets would be maintained by the end of this time period in order not to exceed the upper bound on the accumulated flying hours. For each critical jet that is not maintained within  $\mathcal{T}_t$ , we try to find a day in  $\overline{\mathcal{T}}$  on which it

might be maintained and obtain an approximate value for the penalty that will be incurred for its maintenance. In order to achieve this, we extend the time-expanded network  $D$ . For each day  $\bar{t} \in \overline{\mathcal{T}}$ , we include a node  $u_{\bar{t}}$ . For each node  $(j, t + k, h)$  where  $h$  is greater than the lower bound on the flying hours to be accumulated for jet  $j$  before the next maintenance, we add arcs to nodes  $u_{\bar{t}}$  for all  $\bar{t} \in \overline{\mathcal{T}}$ . Such an arc represents jet  $j$  being maintained on day  $\bar{t}$  and the cost of this arc is equal to the penalty that would have been incurred if jet  $j$  is maintained after accumulating  $h + f(\bar{t} - (t + k))$  hours.

Part of an example network for a single jet is shown in Figure 3.2 where the dotted arcs represent the jet being maintained. The number of days MDP is solved for,  $k$ , is equal to 3 and  $|\overline{\mathcal{T}}| = 3$ . The jet starts day  $t$  with 275 hours accumulated since its last maintenance. Furthermore, it is assumed that  $H$ ,  $H^{min}$  and  $H^{max}$  are 300, 270 and 330. Suppose on day  $t$  there are two itineraries available for the jet denoted by  $i_1$  and  $i_2$  and the flying time of these itineraries are 6 and 10 hours, respectively. Furthermore, suppose the average flying hours  $f$  is 8 hours which is the assumed flying time for jet  $j$  on days  $t + 1$  and  $t + 2$ .

Let  $\mathcal{A}^i$  represent the set of arcs that satisfy itinerary  $i \in \mathcal{I}_t$  and  $\mathcal{A}_{t'}^M$  represent the set of arcs representing a jet being maintained on day  $t'$ . Decision variable  $y_a$  corresponds to the jet flow on arc  $a \in \mathcal{A}$ . MDP can be written as the integer program:



**Figure 3.2:** Part of the network for a single jet

$$(MDP) : \min z = \sum_{a \in \mathcal{A}} c_a y_a \quad (3.1)$$

*s.t.*

$$\sum_{a \in \delta^{out}(n)} y_a - \sum_{a \in \delta^{in}(n)} y_a = \begin{cases} 1 & \text{if } n = r_j \text{ for } j \in J_t^C; \\ -1 & \text{if } n = s_j \text{ for } j \in J_t^C; \forall n \in \mathcal{N} \\ 0 & \text{otherwise.} \end{cases} \quad (3.2)$$

$$\sum_{a \in \mathcal{A}^i} y_a \leq 1 \quad i \in \mathcal{I}_t \quad (3.3)$$

$$\sum_{a \in \mathcal{A}_{t'}^M} y_a \leq cap_{t'} \quad \forall t' \in \mathcal{T}_t \cup \overline{\mathcal{T}} \quad (3.4)$$

$$y_a \geq 0, \text{ integer} \quad \forall a \in \mathcal{A}. \quad (3.5)$$

The objective is to minimize the total cost of flow of jets on the arcs. Constraints

(3.2) are the flow balance constraints for the critical jets. Constraints (3.3) ensure that each itinerary is assigned to at most one critical jet. Constraints (3.4) are the maintenance capacity constraints. Note that the maintenance capacities are parameters here as they already have been determined in the tactical level.

### 3.2.2 Smoothing phase

In the Smoothing Phase (SP), the itineraries of day  $t$  that were not assigned to a critical jet while solving MDP are assigned to non-critical jets with the objective of distributing the number of jets with varying levels of accumulated flying hours evenly. We divide the range of possible accumulated flying hours into buckets where the length of each bucket is equal to the average daily flying time  $f$ . Thus, a bucket also corresponds to a certain number of days until the next maintenance and on each day a jet typically moves from one bucket to the next, each time getting a day closer to maintenance. Having approximately the same number of jets in each bucket, in particular buckets of high accumulated flying hours (i.e. few days away from maintenance), should provide a smooth workflow into the maintenance facility giving MDP ample flexibility to ensure feasibility with respect to capacity.

Let  $B$  denote the set of accumulated hours buckets and  $n_b$  denote the number of days until the next maintenance corresponding to bucket  $b \in B$ . In order to have approximately the same number of jets in each bucket while considering the capacity at the maintenance facility, we penalize deviations of the number of jets in bucket  $b$  from  $cap_{t+n_b}$ , the capacity available on the day these jets will likely be maintained. Note that it might not be possible to distribute the number of jets evenly among the buckets of low accumulated flying hours right after the introduction of new jets. Furthermore, smoothing out an uneven distribution among the buckets of high accumulated flying hours is harder as these jets are only a few days away from maintenance. Thus, the penalty of deviating from the maintenance capacity

gets larger as jets get closer to maintenance.

Let the parameter  $h_{ijb} = 1$  if assigning itinerary  $i$  to jet  $j$  takes it to bucket  $b \in B$  and 0 otherwise. Let  $p_b$  denote the penalty for the deviation of the number of jets in bucket  $b$  from  $cap_{t+n_b}$ . Finally, let the set of critical jets that are in bucket  $b$  be represented by  $J_b^C$ . There are two types of decision variables. The binary decision variable  $w_{ij}$  equals 1 if itinerary  $i \in I_t^U$ , where  $I_t^U$  represents the set of unassigned itineraries of day  $t$ , is assigned to non-critical jet  $j \in J_t^N$  where  $J_t^N$  represents the set of non-critical jets on day  $t$ , and 0 otherwise. The variable  $D_b$  corresponds to the absolute deviation of the number of jets in bucket  $b$  from  $cap_{t+n_b}$ . The integer program solved in the smoothing phase is given as:

$$(SP) : \min z = \sum_{b \in B} p_b D_b \quad (3.6)$$

*s.t.*

$$\sum_{i \in I_t^U} w_{ij} = 1 \quad \forall j \in J_t^N \quad (3.7)$$

$$\sum_{j \in J_t^N} w_{ij} = 1 \quad \forall i \in I_t^U \quad (3.8)$$

$$D_b \geq |J_b^C| + \sum_{i \in I_t^U} \sum_{j \in J_t^N} h_{ijb} w_{ij} - cap_{t+n_b} \quad \forall b \in B \quad (3.9)$$

$$D_b \geq cap_{t+n_b} - |J_b^C| - \sum_{i \in I_t^U} \sum_{j \in J_t^N} h_{ijb} w_{ij} \quad \forall b \in B \quad (3.10)$$

$$w_{ij} \in \{0, 1\} \quad \forall i \in I_t^U, \forall j \in J_t^N \quad (3.11)$$

$$D_b \geq 0, \text{ integer} \quad \forall b \in B. \quad (3.12)$$

The objective is to minimize the total penalty of the absolute deviations of the number of jets in the accumulated hours buckets from maintenance capacity. Constraints (3.7) ensure that each non-critical jet is assigned exactly one itinerary while

constraints (3.8) ensure that each unassigned itinerary of day  $t$  is assigned to exactly one non-critical jet. Finally, constraints (3.9) and (3.10) determine the absolute deviation of the number of jets in bucket  $b$  from  $cap_{t+n_b}$ .

### 3.2.3 Computational Results

We report the results of our solution approach for the 10 instances introduced in Chapter 2. The computations are done over a rolling horizon where the problem for the current day  $t$  in the planning horizon is solved for  $k$  days and only the decisions made for day  $t$  are implemented. When we roll forward one day in the planning horizon, the problem is resolved for  $k$  days starting from day  $t + 1$  with the newly available information about the itineraries on day  $t + 1$ . The code is written in C++ and the IP models are built and solved with Concert 2.1 in ILOG CPLEX 9.1 using 2.4 GHz AMD 250 processors with 4 GB of RAM.

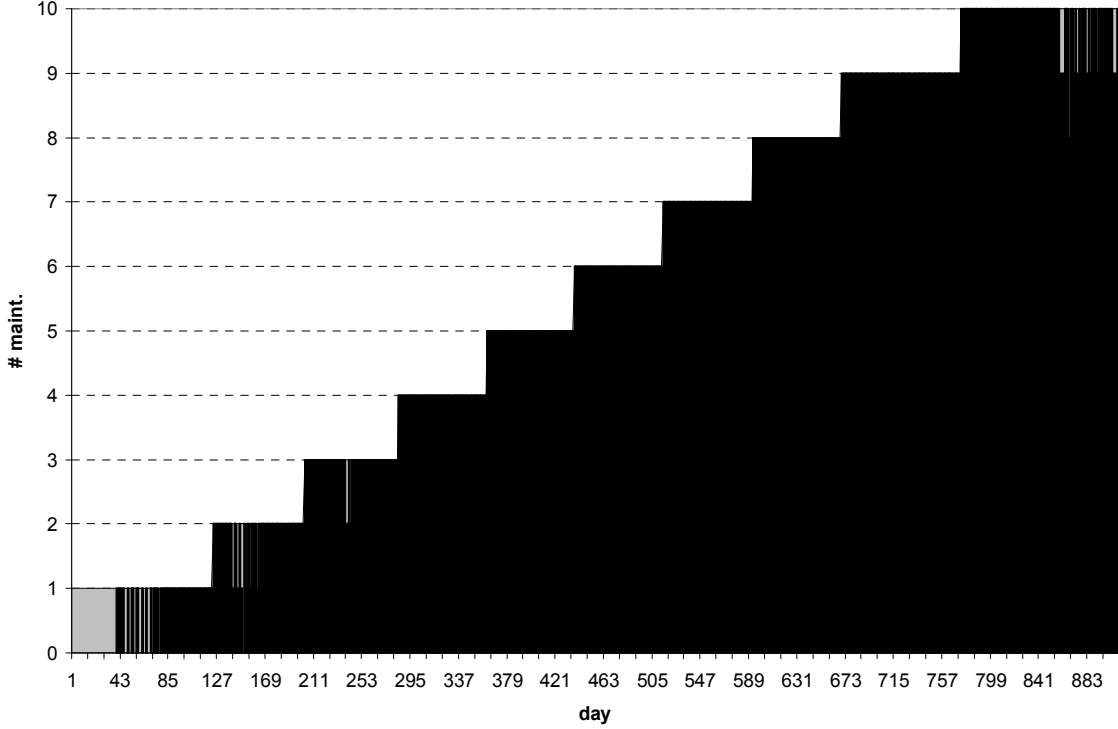
The experimental settings used during the computations are as follows. We assume that all itineraries have a flight time equal to a random integer between 8 and 12 hours. The jets in the fleet are assumed to be evenly distributed among the bases. If a jet is scheduled for maintenance on day  $t$ , its total flying time on days  $t$  and  $t + 1$  are incremented by the flying time to the maintenance facility from its base and the flying time from the maintenance facility to its base, respectively. Initially, the average flying time that is used for unknown itineraries in the future is set to be 10 hours. As we move further in the planning horizon, this average is updated with a moving average with exponential smoothing. The problem parameters related to maintenance are the same as before, i.e.  $H$ ,  $H^{min}$  and  $H^{max}$  are equal to 300, 270 and 330, respectively. The maintenance capacities presented in Section 2.2.3 that were obtained with the optimization-based local search algorithm within 2 hours are used during the experiments. The cost of accumulating more than the upper bound on the accumulated flying hours before the next maintenance,  $C$ , is set to be larger than

$30^2|J|$ , which is the largest total penalty for feasible maintenance. Finally,  $p_b$ , which is used in the smoothing phase to penalize the deviations of the number of jets in accumulated flying hour bucket  $b$  from the capacity is set to be  $2^{(\lceil \frac{H}{f} \rceil - n_b)}$  where  $n_b$  is the number of days until maintenance for a jet which is bucket  $b$ . After experimenting with different values for the look-ahead length  $k$  while solving MDP,  $k = 10$  was found to be the best choice. Also, choosing  $|\overline{\mathcal{T}}| = 10$ , which is the length of the additional time period considered while solving MDP to capture the cost of not maintaining a jet, is enough to ensure all critical jets would be maintained feasibly within the given time period.

Figure 3.3 and Figure 3.4 show the number of jets maintained on each day for instances  $(288J, 2D)$  and  $(288J, 2M)$ , respectively. The area shown by the lighter color represents the maintenance capacity while the darker area to the front gives the number of jets maintained on each day. As can be seen from the figures, the number of jets maintained on each day closely follows the maintenance capacity for instance  $(288J, 2D)$  while there is unused capacity during the first year for instance  $(288J, 2M)$ . This is expected since the maintenance capacity needs to be increased to higher levels earlier when jets are introduced in larger batches. Capacity usage rate is 99% and 86% for instances  $(288J, 2D)$  and  $(288J, 2M)$  respectively which is large enough to indicate a good match between the requirements of operational planning and the output of tactical planning.

Figure 3.5 and Figure 3.6 show the histograms of the accumulated flying hours between successive maintenance activities for instances  $(288J, 2D)$  and  $(288J, 2M)$ , respectively. As can be seen from the figure, the majority of the time jets are scheduled for maintenance within the target accumulated flying hours. It can also be observed that the distribution of the deviations around the target is more uniform for instance  $(288J, 2M)$  while most of the deviations for instance  $(288J, 2D)$  are above the target. This result is due to the fact that maintenance capacity is very tight for instance





**Figure 3.3:** Maintenance chart for instance (288J,2D)

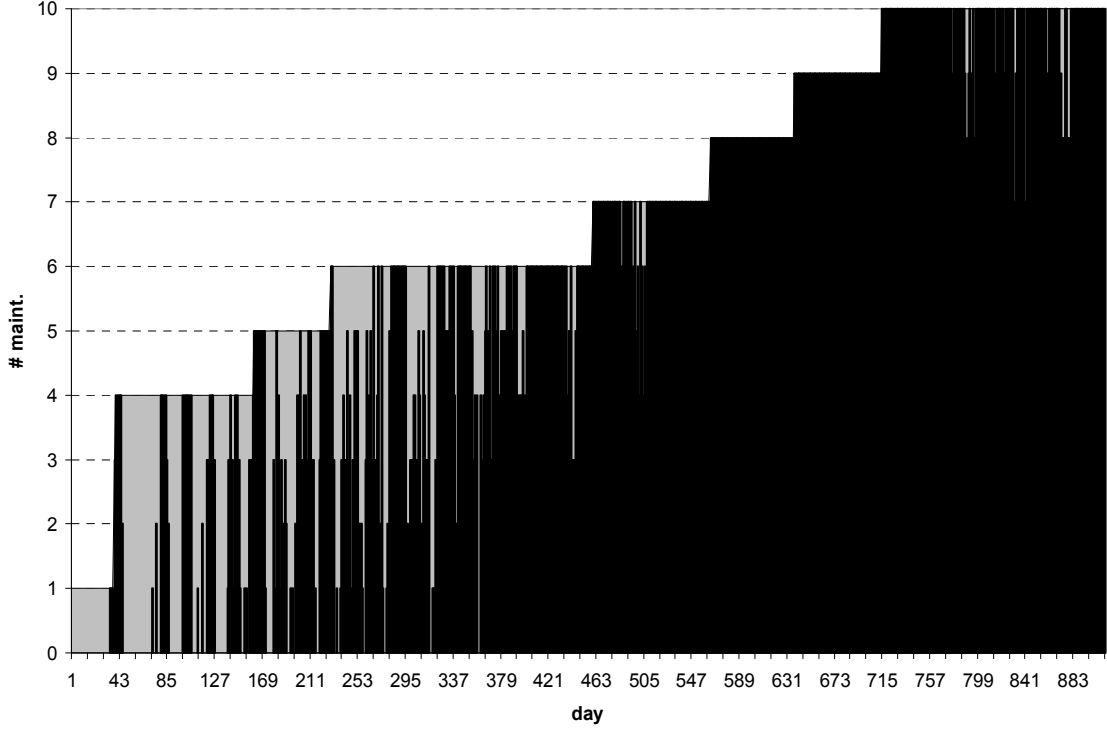
(288J, 2D) and thus, it is almost impossible to maintain some jets earlier using the unused available capacity when it is obvious that some other jets are likely to be maintained late.

Table 3.1 summarizes the results for all 10 instances. The first column shows the total number of times maintenance is performed whereas the second column represents the percentage of total capacity that is used for maintenance during the planning horizon. The third and fourth columns show the average and the standard deviation of the accumulated flying hours between maintenance activities respectively. The fifth column corresponds to the percentage of maintenance activities that violate the maintenance rules. Finally, the last column represents the average time it takes to solve the operational maintenance planning problem.

From Table 3.1, it can be seen that capacity usage rate is above 97% for almost all the instances. As mentioned before, the capacity usage rate for instances in which jets

**Table 3.1:** Summary results for the operational maintenance planning problem

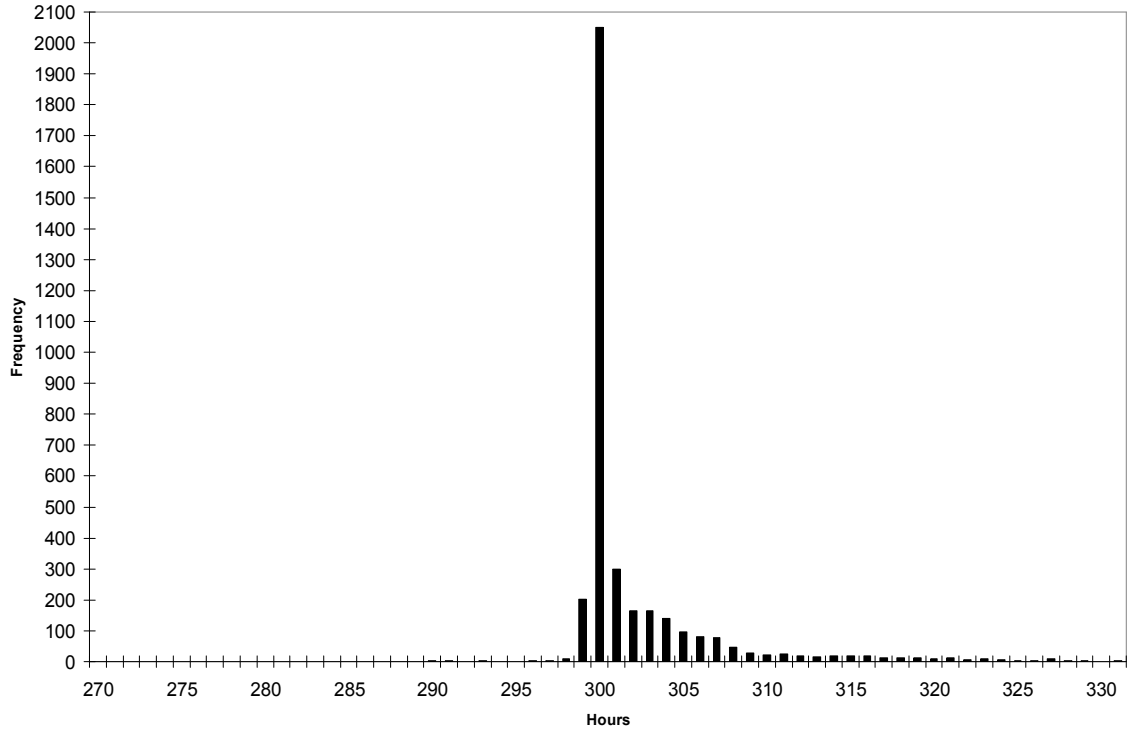
Instance	No. of maint.	% cap. used	Avg. acc. hrs.	Std. dev. of acc. hrs.	% infeasible maint.	Avg. time (sec)
(288J, 2D)	3618	98.91	302.09	5.11	0.53	5.14
(288J, 1W)	3634	98.97	302.05	5.03	0.33	5.43
(288J, 2W)	3661	98.23	301.87	4.75	0.11	5.5
(288J, 1M)	3725	97.08	301.12	3.44	0	5.87
(288J, 2M)	3841	86.68	300.34	2.7	0	5.93
(480J, 1D)	6000	99.83	301.87	3.44	0	11.72
(480J, 1W)	6037	99.92	301.82	3.37	0	11.78
(480J, 2W)	6086	99.07	301.68	3.36	0	11.84
(480J, 1M)	6178	98.28	301.03	3.15	0	11.94
(480J, 2M)	6373	88.34	300.56	2.41	0	12.02



**Figure 3.4:** Maintenance chart for instance (288J,2M)

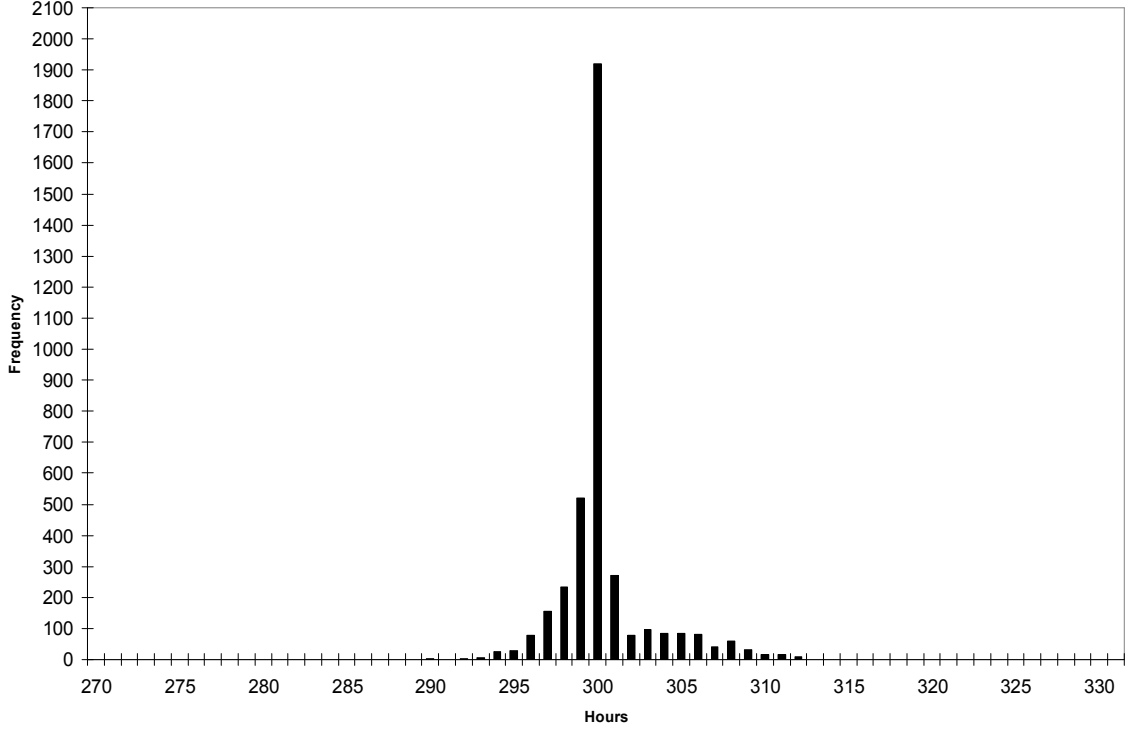
are introduced with the largest batches and least frequently is lower since maintenance capacity has to be increased quickly to larger levels in order to accommodate the early maintenance requirements of the batches arriving together. The high capacity usage rate together with a rate of compliance with the rules above 99% indicate a good match between the output of tactical planning and the requirements of operational planning. It can also be observed that with the given capacity we can achieve our maintenance objectives since average accumulated flying hours between successive maintenance activities is very close to the target 300 hours for all instances while the standard deviation is at most approximately 5 hours. Furthermore, the average time it takes to solve the operational maintenance planning problem is quite small even for the large instances with 480 jets.

The average and the variance of accumulated flying hours between successive



**Figure 3.5:** Histogram of the accumulated flying hours for instance (288J,2D)

maintenance activities improve as jets are introduced in larger batches and less frequently. This is due to the fact that capacity is increased earlier for these instances and with more capacity available, some jets can be maintained a little earlier to avoid maintaining other jets much later. Another advantage of capacity increasing earlier for these instances is in the lower rate of maintenance activities violating the rules. However, these improvements are not large enough to justify the increase in the total number of maintenance activities resulting in larger costs. Thus, introducing jets into the fleet in smaller batches and more frequently seems to be better for operational planning purposes as well.



**Figure 3.6:** Histogram of the accumulated flying hours for instance (288J,2M)

### 3.3 *Integration with Flight Scheduling*

In this section, we present a framework for capturing the interaction between operational maintenance decisions and flight scheduling. Our approach to making operational maintenance decisions needs to be slightly modified to handle the interaction with flight scheduling.

*Making maintenance decisions in advance:* Typically, transportation requests are received a few days in advance and it is the task of the online flight scheduling algorithm to decide whether such a request can be accommodated. Therefore, if we try to make maintenance decisions for a given day only when the flight schedule for that day has been determined by the off-line flight scheduling algorithm, it might not be possible to implement the preferred maintenance decisions while still accommodating all transportation requests for that day. As a consequence, especially if this happens

frequently, we may not be able to satisfy the maintenance mandates. To prevent this from happening, we make maintenance decisions a few days in advance. That is, on a given day  $t$  we make maintenance decisions for day  $t + l$ , where  $l$  is a parameter specifying how many days in advance maintenance decisions are made. (Note that the jets that will be maintained on days  $t, \dots, t + l - 1$  have already been determined and these decisions are not changed.) By making maintenance decision a few days in advance, the online flight scheduling algorithm is aware of any reduced capacity due to maintenance decisions.

*Updating maintenance decisions:* Even if maintenance decisions are made a few days in advance, it may happen that preferred maintenance decisions for day  $t + l$  cannot be implemented because a large number of transportation requests has already been accepted for day  $t + l$  (and these commitments have to be honored). In this situation, it is necessary to adjust the decisions for the jets that cannot be maintained on day  $t + l$ . These jets are scheduled for maintenance as early as possible after day  $t + l$  (while ensuring that accepted requests on a day can always be accommodated). Note that by adjusting the day of maintenance for these jets forward in time maintenance capacity becomes available on day  $t + l$ . Thus, it may be a good idea to maintain some other jets on day  $t + l$ . To evaluate that possibility, we resolve MDP (and continue to iterate this process as long as maintenance decisions are adjusted due to the requirements of flight scheduling). Once all the maintenance decisions for day  $t + l$  can be feasibly implemented, we move to the smoothing phase.

Thus, the daily early morning decision process on each day  $t$  is as follows. First, itineraries are constructed for all accepted transportation requests for day  $t$  using the off-line flight scheduling algorithm. After the itineraries are constructed, MDP is solved to determine which jets are to be maintained on day  $t + l$  as well as to assign itineraries to the critical jets on day  $t$ . Let the set of jets to be maintained on day  $t + l$  be denoted by  $\mathcal{J}^M$ . For each jet in  $\mathcal{J}^M$ , we check whether it can be

maintained while still accommodating all requests that have already been accepted for day  $t + l$ . If the jet can be maintained feasibly, the decision for this jet is finalized and the online flight scheduling algorithm is made aware of that maintenance. If the jet cannot be feasibly maintained, the earliest day after  $t + l$  is found where the jet can be maintained and the decision to maintain the jet on that day is finalized. If any of the maintenance decisions for the jets in  $\mathcal{J}^M$  is adjusted, MDP is solved again to see if the maintenance capacity on day  $t + l$  that has become available can be used effectively. The process continues until all maintenance decisions for day  $t + l$  can feasibly be implemented. After that the smoothing algorithm is called to assign itineraries to the non-critical jets.

### 3.3.1 Case Study

In this section, we describe a case study based on simulated operations at DayJet Corporation, a startup PSOD air transportation provider. The emphasis of this case study is on evaluating the operational maintenance decisions in the presence of flight scheduling. The scenario in the case study represents an environment in which the travel demand is highly variable from day to day in order to stress test the models and solution approaches. A total of 174 jets are introduced into the fleet over a two year planning horizon with an average of 1 month between jet arrivals according to the contract with the jet manufacturer. The average number of jets arriving per month is lower over the first year. Furthermore, the growth rate of travel demand is also lower during the first year and gradually increases during the second year.

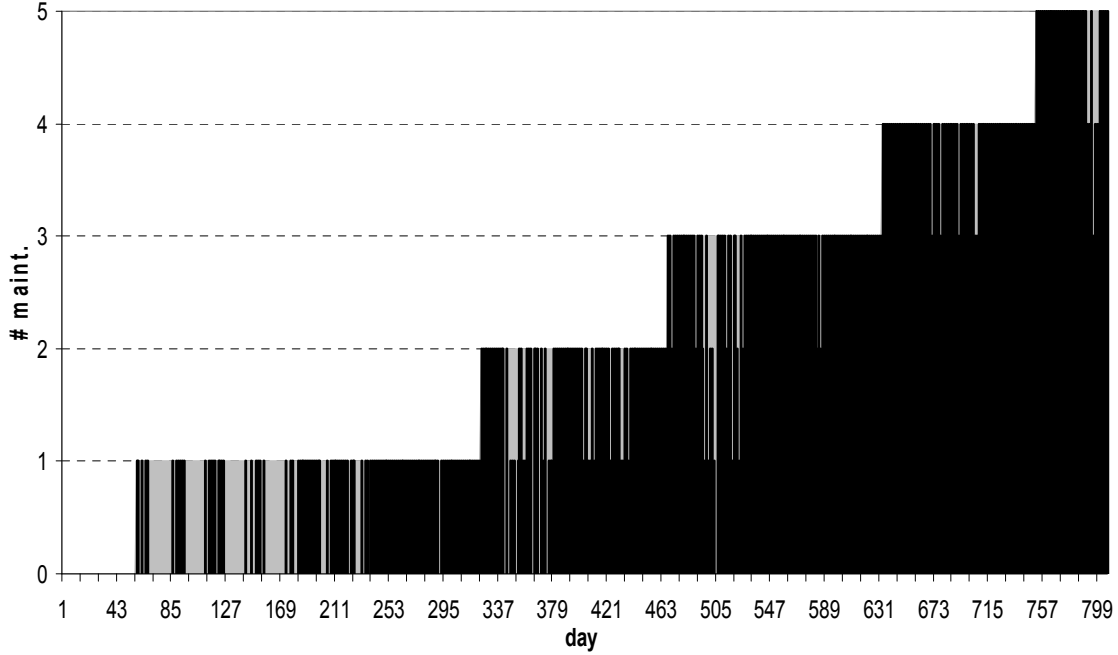
At the tactical level, we determine the capacity at the maintenance facility using our optimization-based local search algorithm with a time limit of 2 hours. The values of the local search parameters determining the neighborhood sizes are the same as the ones given in Chapter 2. The average flying time,  $f$ , gradually increases from 5 hours to 10 hours during the planning horizon in accordance with the scenario. The

integrated framework of operational maintenance planning and flight scheduling is implemented as explained in Section 3.3 on each day of the planning horizon. The framework is tested in a simulation environment where transportation requests are generated using an agent-based model developed by DayJet Corporation. Furthermore, online and off-line flight scheduling are performed using algorithms and decision support tools developed by DayJet as well. The values for the parameters of operational maintenance planning such as the look-ahead length  $k$  and the penalties for MDP and SP are chosen to be the same as the ones given in Section 3.2.3. Historical data regarding the travel requests indicates that requests arrive on average two days ahead of their travel date. Thus accordingly, we set the parameter  $l = 2$  corresponding to the number of days maintenance decisions are made in advance. The code is written in C++ and the IP models are built and solved with Concert 2.6 in ILOG CPLEX 11.1 using a 2.5 GHz Intel Core2 Q9300 processor with 4 GB of RAM.

Figure 3.7 shows the number of jets maintained on each day whereas Figure 3.8 shows the histogram of the accumulated flying hours before maintenance. The number of jets maintained on each day still closely follows the maintenance capacity, especially during the second year. Since the growth rate of the fleet is small during the first year and the jets need to accumulate a minimum number of flying hours before maintenance, there are days where no jet can be maintained. The histogram shows that maintenance is still mostly done after accumulating 300 flying hours which is the target value. The standard deviation of the accumulated flying hours is larger compared to the results presented for operational maintenance planning in Section 3.2.3. One of the reasons for the larger standard deviation is that the travel demand is highly variable from day to day. While planning for maintenance 2 days in advance, the *anticipated* (average) flying times might seem take a jet very close to the target  $H$  before maintenance although this target might be more likely to be missed with

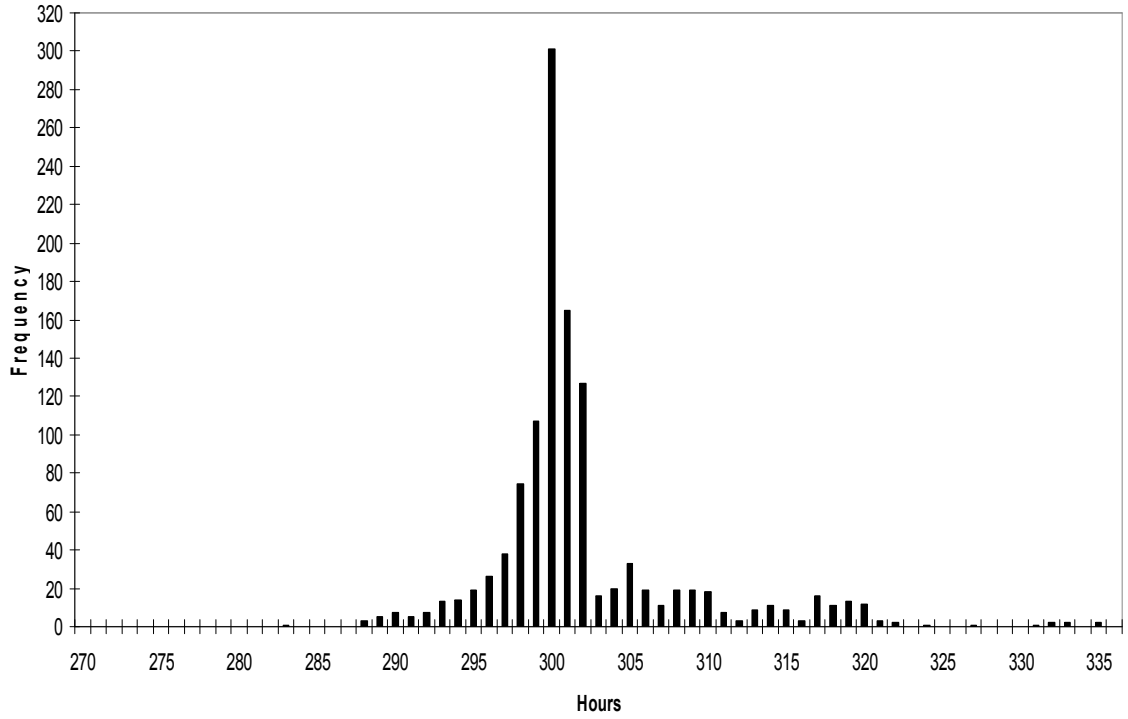


the *actual* realized flying times due to the high variation in the travel demand. Furthermore, sometimes maintenance has to be postponed in order to accommodate the requests that have already been accepted, which in turn results in jets accumulating more flying hours than planned.



**Figure 3.7:** Maintenance chart for the simulated case study

Table 3.2 summarizes the results for the simulated case study. The first row shows the total number of times maintenance is performed during two years. The second row represents the percentage of total capacity used for maintenance during the planning horizon. The average and the standard deviation of the accumulated flying hours before maintenance are given in the third and the fourth rows, respectively. The fifth row corresponds to the percentage of maintenance activities that violate the maintenance rules. Finally, the last two rows give the percentage of maintenance activities that have to be postponed due to the impact of flight scheduling and the average number of days they are postponed for.



**Figure 3.8:** Histogram of the accumulated flying hours for the simulated case study

The results in Table 3.2 demonstrate that we obtain very good results using the proposed framework of the integration of operational maintenance planning with flight scheduling for the operations of DayJet. A capacity usage rate of 90% and less than 2% violations of the maintenance rules indicates a good fit between the maintenance capacity and the requirements of operational planning. The slight increase in the infeasibility rate, over the earlier experiments in Section 3.2.3, can be attributed to the need to postpone a portion of maintenance activities in order to accommodate accepted transportation requests. However, as can be seen from Table 3.2, only 6% of the maintenance activities have to be postponed due to the impact of flight scheduling and on average they can be maintained the next day. Furthermore, we can still achieve our maintenance objectives since the average of the accumulated flying hours before maintenance is very close to the target 300 hours and the standard deviation is around 7 hours. Thus, our solution approaches are robust enough to handle high variability

**Table 3.2:** Summary results for the simulated case study

No. of maint.	1175
% cap. used	89.46
Avg. acc. hrs.	301.95
Std. dev. of acc. hrs.	6.53
% infeasible maint.	1.96
% maint. postponed	6.04
Avg. no. of days postponed	1.26

in travel demand.

## CHAPTER IV

### BASE LOCATION AND FLEET ALLOCATION

#### 4.1 *Introduction*

Flight scheduling is one of the most important operations for PSOD air transportation. Therefore, decision making at all levels of planning that affect flight scheduling must be given careful consideration. At the strategic level, decisions are made regarding which airports to serve, i.e. at which airports to pickup and drop-off passengers. Once the airports to be served are determined, base location and fleet allocation decisions, i.e. which airports are to be operated as bases and how many jets in the fleet will be assigned to each base, are made at the tactical level. Finally, given a set of bases and the number of jets at each base, flight schedules can then be constructed on a daily basis at the operational level.

To reach profitability, a PSOD air transportation business needs to grow its demand and satisfy that demand in a cost-efficient way. The base location and fleet allocation decisions affect both travel demand and efficiency. Daily operational costs are proportional to the time traveled by the jets. As flight legs without passengers, called *deadheads*, bring no revenue but incur operational costs, it is desired to minimize their occurrence. Strategically allocating jets to airports in close proximity to locations that are likely to generate large volumes of demand can minimize deadheads and also increase the likelihood of being able to accommodate more transportation requests.

The base location and fleet allocation decisions for PSOD air transportation are made over a planning horizon during which the business may experience changes (typically growth) in both demand and fleet size. Growing demand is accomplished

in part by opening new markets, i.e. serving additional airports. When the number of airports served grows, the number of bases needs to grow as well for efficient operations. At time points corresponding to the arrival of new jets, one has to decide whether to open new bases and, if so, where to open them considering the current demand, and how to allocate the current fleet amongst the bases to best satisfy this demand. In order to solve this multi-period problem, the information needed at each decision point is the the number of jets in the fleet and and the demand forecasted for the upcoming period. We consider two possibilities for the forecasted demand in each period. A realistic setting is to assume that demand information is updated at the beginning of each period, i.e. when new jets arrive. With this assumption, we determine the base location and fleet allocation decisions period by period making the best decisions given the information available at that time. In order to assess the quality of this dynamic decision making, we consider the case where we assume that perfect demand information is available for the entire planning horizon and thus we can make decisions for all time periods simultaneously.

In order to make good base location and fleet allocation decisions, it is necessary to incorporate as much information as possible about travel demand (in the form of transportation requests) and operational flight scheduling. We investigate two approaches that capture such information with different levels of detail. In the first approach, which is referred to as the *basic approach*, the information about travel demand and flight scheduling is captured simply as the number of jets that is demanded at each airport. Using this as an input, we then solve a traditional facility location problem to make the base location and fleet allocation decisions. We next develop a model that works directly with transportation requests and integrates a simplified version of flight scheduling with the base location and fleet allocation decisions. We refer to this second approach, which captures travel demand and flight scheduling in more detail, as the *integrated approach*.

## 4.2 *Problem Description*

The base location and fleet allocation problem is to determine where to open (new) bases and how to allocate the current fleet among these bases over a planning horizon during which the travel demand and the fleet size might change. As demand changes, the locations of some of the previously opened bases might lose their appeal and it might be necessary to update the allocation of the jets to the bases. However, due to the large investment made to open a base we assume that once a base is opened it will stay open in the future, at least during the planning horizon that is considered. This large investment also warrants allocating at least a certain number of jets to a base. Furthermore, there is an upper bound on the number of jets that can be allocated to a base due to physical space limitations.

The objective while making the base location and fleet allocation decisions is to minimize the expected operational costs. Since operational costs are proportional to the time traveled by the jets, total flying time is used as a substitute for the operational costs, i.e. the objective is to minimize the expected total flying time. In order to make it possible to compare different approaches to capturing information about flight scheduling possible, we assume that the number of bases to be opened at each decision point is given. In reality, it is more likely that a trade-off needs to be made between the fixed cost of locating a facility and the operational cost of satisfying the demand of customers. Once the approach for making the base location and fleet allocation decisions is determined, the fixed cost associated with opening a base can be incorporated into the model.

We assume that demand information is updated at regular intervals during the planning horizon and we make the base location and fleet allocation decisions at the beginning of each time period. Let  $\mathcal{T}$  and  $\mathcal{A}$  denote the set of time periods and the set of all airports, respectively. Let  $o^t = \{o_a^t : a \in \mathcal{A}\}$  denote the set of binary decision variables that equal 1 if a base is operating at airport  $a$  in time period  $t$ . The set of

integer variables  $y^t = \{y_a^t : a \in \mathcal{A}\}$  corresponds to the number of jets allocated to each airport in time period  $t$ . The minimum and the maximum number of jets to be allocated to a base are denoted by  $n_{min}^j$  and  $n_{max}^j$ , respectively. Let  $n_t^B$  represent the number of bases to be in operation and  $\mathcal{J}_t$  denote the set of jets that are active in time period  $t$ . Finally, let  $\bar{o}_a^{t-1}$  denote the parameter that equals 1 if a base was operating at airport  $a$  in time period  $t$  and 0 otherwise. Then, the single period problem to be solved at time period  $t$  ( $SPP^t$ ) is given as

$$(SPP^t) : \min z = \mathbb{E}[C(o^t, y^t)] \quad (4.1)$$

s.t.

$$o_a^t \geq \bar{o}_a^{t-1} \quad \forall a \in \mathcal{A} \quad (4.2)$$

$$\sum_{a \in \mathcal{A}} o_a^t = n_t^B \quad (4.3)$$

$$y_a^t \geq n_{min}^j o_a^t \quad \forall a \in \mathcal{A} \quad (4.4)$$

$$y_a^t \leq n_{max}^j o_a^t \quad \forall a \in \mathcal{A} \quad (4.5)$$

$$\sum_{a \in \mathcal{A}} y_a^t = |\mathcal{J}_t| \quad (4.6)$$

$$o_a^t \in \{0, 1\} \quad \forall a \in \mathcal{A} \quad (4.7)$$

$$y_a^t \geq 0, \text{ integer } \forall a \in \mathcal{A} \quad (4.8)$$

$$(o^t, y^t) \in Q_t \quad (4.9)$$

where  $C(o^t, y^t)$  represents the total operational cost (total flying time) associated with the decision set  $(o^t, y^t)$  and  $Q_t$  corresponds to the set of constraints that capture the information about demand and flight scheduling which are handled differently depending on the level of detail considered. The objective is to minimize the expected operational costs. Constraints (4.2) make sure that a base that was operating at an earlier time period is not closed. Constraints (4.3) ensure that the total number of bases operating in time period  $t$  is equal to the predetermined number. Constraints

(4.4) and (4.5) consider the minimum and maximum number of jets to be allocated to a base, respectively. Finally, constraints (4.6) make sure that the total number of jets allocated to the bases is equal to the current size of the fleet in time period  $t$ .

When we assume that demand information is available for the entire planning horizon, we can make decisions for all time periods simultaneously using a multi-period model (*MPP*). The objective of *MPP* is to minimize the total present value of the expected operational costs. Let  $\alpha$  represent the discount rate used for future costs, i.e.  $\alpha = \frac{1}{1+i}$  where  $i$  corresponds to the interest rate per time period. *MPP* is

$$(MPP) : \min z = \sum_{t \in \mathcal{T}} \alpha^{t-1} \mathbb{E}[C(o^t, y^t)] \quad (4.10)$$

*s.t.*

$$(o^t, y^t) \text{ satisfy (4.2) -- (4.9) } \forall t \in \mathcal{T}. \quad (4.11)$$

### 4.3 *Basic approach*

In the basic approach, we use a traditional facility location model where the information about demand and flight scheduling is captured simply as the number of jets “demanded” at each airport. The idea behind finding the number of jets demanded at each airport is that an airport needs jets to avoid rejection of transportation requests or deadheads to satisfy these requests. Thus, given a total number of available jets, the demand of each airport is obtained by distributing this total number proportionally according to the relative likelihood of jets being needed at these airports.

To determine the relative likelihood of jets being needed at an airport, we consider forecasted transportation requests and flight scheduling knowledge. For a particular set of daily transportation requests, we first try to incorporate high level flight scheduling information. One of the important strategies of flight scheduling is the aggregation of transportation requests. Aggregation occurs when the passengers associated with

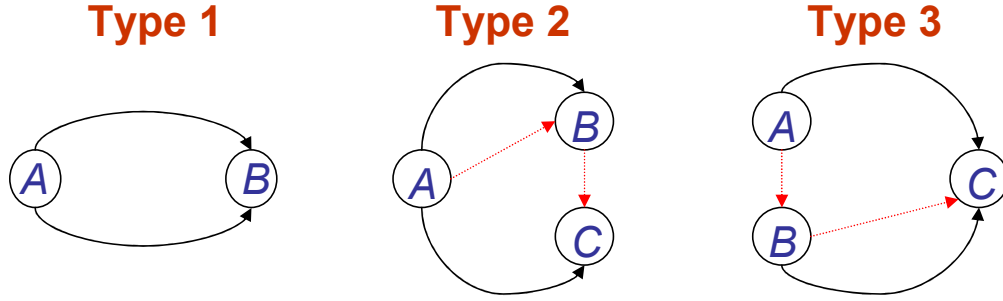


different transportation requests share a jet. In PSOD air transportation where the time of the passengers is valuable, the number of intermediate stops is limited (most likely at most one) and this must be taken into account while aggregating requests. For example, if each request is to be transported from its origin to its destination with at most one intermediate stop, then the transportation requests to be aggregated should have a common origin and/or destination. In order to incorporate this high level information about flight scheduling, we analyze the transportation requests to find potential aggregations.

For ease of description we explain possible aggregations for the case where at most one intermediate stop is allowed. In this case, there are three types of possible aggregations: (*Type 1*) aggregating transportation requests with the same origin and the same destination, (*Type 2*) aggregating transportation requests with the same origin and different destinations, and (*Type 3*) aggregating transportation requests with the same destination and different origins. Figure 4.1 summarizes the possible aggregations for two transportation requests with at most one intermediate stop. In all aggregations, there are two main considerations: whether the seating capacity of a jet is exceeded on a flight leg, and whether the latest arrival times of all the requests that are aggregated can still be met with the updated departure times.

*Type 1 Aggregation:* Both transportation requests fly directly from their origins to destinations. Thus, they are combined into a single request with origin  $A$  and destination  $B$ . The earliest departure time of the aggregate request is set to be the maximum of the earliest departure times of the separate requests and the latest arrival time of the aggregate request is set to be the minimum of the latest arrival times of the separate requests.

*Type 2 Aggregation:* The requests share a flight leg from airport  $A$  to airport  $B$ . The passengers of the request with destination  $B$  are dropped off at airport  $B$  and the passengers of the request with destination  $C$  continue to fly from airport  $B$  to airport



**Figure 4.1:** Different ways of aggregating transportation requests with at most one intermediate stop

*C*. Thus, the requests are combined into a single request with origin *A* and destination *C*. The earliest departure time of the aggregate request is set to be the maximum of the earliest departure times of the separate requests and the latest arrival time of the aggregate request is set to be the latest arrival time of the request that is destined for airport *C*.

*Type 3 Aggregation:* The passengers of the request with origin *A* are flown to airport *B* to pick up the passengers of the request with origin *B*. Then, they share the flight leg from airport *B* to airport *C*. Thus, the requests are combined into a single request with origin *A* and destination *C*. The earliest departure time of the aggregate request is set to be the earliest departure time of the request that originates at *A* and the latest arrival time of the aggregate request is set to be the minimum of the latest arrival times of the separate requests.

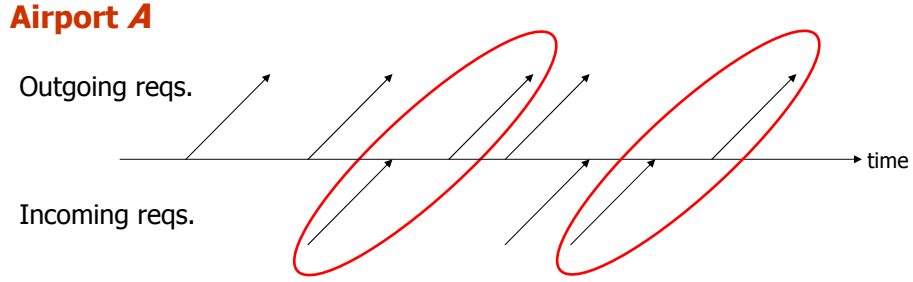
Once the high level information about flight scheduling is incorporated into the travel requests, the relative likelihood of a jet being needed at an airport is found using two different schemes: (i) an activity-based scheme, and (ii) an imbalance-based scheme.

*Activity-based demand scheme:* This demand scheme is based on the idea that an

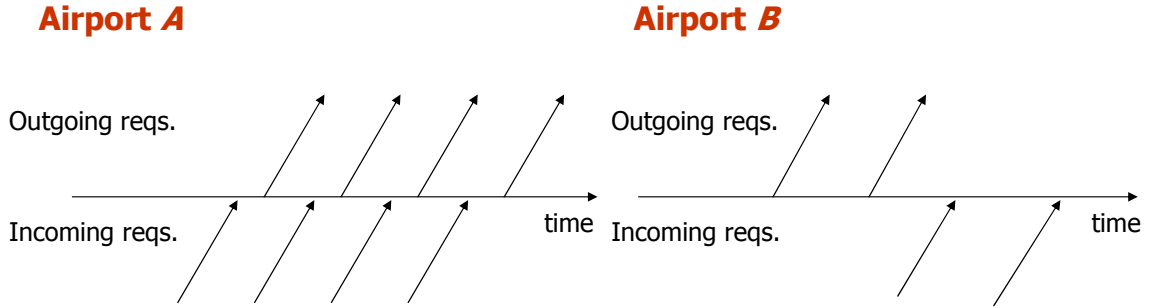
airport with more activity is likely to need more jets where the activity for an airport is taken to be the total number of outgoing transportation requests. This demand scheme tries to capture as much demand as possible since the more jets are located at airports with large activity, the more likely it is to accommodate the transportation requests originating at those airports. The relative likelihood of jets being needed at an airport is obtained by taking the ratio of the activity of this airport to the total activity amongst all airports.

*Imbalance-based demand scheme:* This demand scheme is based on the idea that an airport with more imbalance between the incoming and outgoing requests during the day is likely to need more jets to avoid deadheads. The imbalance of incoming and outgoing requests is defined to be the number of outgoing requests that cannot be matched with an incoming request. The larger this imbalance is for an airport, the more jets will be needed to deadhead there to accommodate the outgoing requests. Thus, deadheads can be avoided by having more jets available at such airports. The imbalance of incoming and outgoing requests can be found as shown in Figure 4.2 given arrival and departure times for the incoming and outgoing requests, respectively. In this example, two outgoing requests in the morning and one outgoing request in the afternoon cannot be matched with an incoming request and thus, the imbalance for this airport is 3. The relative likelihood of jets being needed at an airport is obtained in a way similar to the activity based scheme by taking the ratio of the imbalance of each airport to the sum of these imbalance values amongst all airports.

The difference between these two different demand schemes can be seen more clearly from the example depicted in Figure 4.3. As shown in the figure, airport *A* has more total outgoing requests and thus more activity. However, all the outgoing requests can be balanced with another incoming request. On the other hand, airport *B* has less activity but none of its outgoing requests can be balanced with another incoming request. The activity-based demand scheme would give more weight to



**Figure 4.2:** Finding the imbalance between incoming and outgoing requests at an airport



**Figure 4.3:** Example depicting the difference between the demand schemes

airport *A* with the hope of accommodating more transportation requests and thus increasing revenue whereas the imbalance-based demand scheme would give more weight to airport *B* with the hope of decreasing the deadhead costs.

Given a set of daily transportation requests, the procedure to find the demand of jets at each airport can be summarized in Algorithm 4.1.

#### 4.3.1 Solution Approach

The number of jets demanded at each airport is uncertain since the travel demand is not known with certainty. Thus, the resulting problem is a two-stage stochastic

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**Algorithm 4.1** FINDING THE DEMAND OF EACH AIRPORT

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- 1: Search for possible aggregations between the transportation requests.
  - 2: **for** each airport **do**
  - 3:     Find the relative likelihood of a jet being needed at that airport to avoid deadheads or rejection of requests using one of the demand schemes.
  - 4:     Obtain the demand of the airport by multiplying the total number of available jets with the likelihood of a jet being needed at that airport.
  - 5: **end for**
- 

facility location problem. The first stage consists of determining the locations of the bases and how many jets to allocate to each base. These decisions have to be made before the demand of each airport is known. In the second stage, once the demands of the airports are known we need to determine how the demand of each airport will be satisfied.

A generic formulation for a two-stage stochastic problem is

$$z^* = \min_{x \in X} \{z(x) := \mathbb{E}_P G(x, W)\} \quad (4.12)$$

where  $x$  and  $X$  represent the first stage decision variables and the first stage feasible set, respectively.  $W$  denotes a random vector with known probability distribution  $P$ . In realistic applications, it is not practical to calculate the expected value  $\mathbb{E}_P G(x, W)$  because of the prohibitively large number of scenarios. Thus, several sampling based approaches have been proposed. Sampling based approaches can be classified into two main groups: interior and exterior sampling methods. Interior sampling methods aim to solve the original problem 4.12 by sampling whenever the algorithm requires a value of  $z(\cdot)$  or subgradient information for  $z(\cdot)$  at some point  $x$ . Examples of such methods are the L-shaped algorithm (Van Slyke and Wets [43]), stochastic decomposition (Higle and Sen [19]) and importance sampling (Infanger [20]). In exterior sampling methods, a sample is selected a priori and a corresponding approximation to  $z(\cdot)$  is defined from this sample. Sample average approximation (SAA) is an exterior sampling method in which the expected value function  $\mathbb{E}_P G(x, W)$  is approximated

by the sample average function  $\sum_{n=1}^N G(x, W^n)/N$  where  $W^1, W^2, \dots, W^N$  represents a random sample of  $N$  realizations of the random vector  $W$  (Kleywegt et al. [22], Mak et al. [24], Norkin et al. [30]).

Let the set of constraints (4.2) - (4.8) be denoted by  $S^t$ , i.e. the first stage feasible set, and  $t_{ab}^f$  denote the flying time from airport  $a$  to airport  $b$ . For a given set of transportation requests and the number of jets demanded at each airport (denoted by  $d_a^t$ ) derived from these requests, the deterministic facility location model corresponding to the single period problem  $SPP^t$  can be given as follows. The integer decision variables  $x_{ab}^t$  represent the number of jets from airport  $a$  used to satisfy the demand of airport  $b$ .

$$\min z = \sum_{a \in \mathcal{A}} \sum_{b \in \mathcal{A}} t_{ab}^f x_{ab}^t \quad (4.13)$$

s.t.

$$(o^t, y^t) \in S^t \quad (4.14)$$

$$\sum_{b \in \mathcal{A}} x_{ab}^t \leq y_a^t \quad \forall a \in \mathcal{A} \quad (4.15)$$

$$\sum_{b \in \mathcal{A}} x_{ba}^t \geq d_a^t \quad \forall a \in \mathcal{A} \quad (4.16)$$

$$x_{ab}^t \geq 0, \text{ integer } \forall a, b \in \mathcal{A} \quad (4.17)$$

We use bold face to distinguish the random vectors from their particular realizations. Let  $\mathbf{W}$  denote the random vector for the transportation requests whereas  $W$  represents a particular realization of daily transportation requests. Let  $d_a^t(W)$  denote the number of jets demanded at airport  $a$  derived from  $W$ . Then,  $SPP^t$  with the basic approach is:

$$\min z = \mathbb{E}[C(o^t, y^t, \mathbf{W})] \quad (4.18)$$

*s.t.*

$$(o^t, y^t) \in S^t, \quad (4.19)$$

where  $C(o^t, y^t, W)$  is the optimal value of the following problem:

$$\min z = \sum_{a \in \mathcal{A}} \sum_{b \in \mathcal{A}} t_{ab}^f x_{ab}^t \quad (4.20)$$

*s.t.*

$$\sum_{b \in \mathcal{A}} x_{ab}^t \leq y_a^t \quad \forall a \in \mathcal{A} \quad (4.21)$$

$$\sum_{b \in \mathcal{A}} x_{ba}^t \geq d_a^t(W) \quad \forall a \in \mathcal{A} \quad (4.22)$$

$$x_{ab}^t \geq 0, \text{ integer } \forall a, b \in \mathcal{A} \quad (4.23)$$

We use the SAA method to solve our two-stage stochastic facility location problem. For time period  $t$ , we generate  $N$  realizations of daily transportation requests. Using these  $N$  different realizations, we find the corresponding demand values for each airport with Algorithm 4.1. The expectation  $\mathbb{E}[C(o^t, y^t, \mathbf{W})]$  is approximated by the sample average function  $\frac{1}{N} \sum_{n=1}^N C(o^t, y^t, W^n)$  and the “true” problem (4.18) - (4.19) is approximated by the problem

$$\min \hat{z}_N = \frac{1}{N} \sum_{n=1}^N C(o^t, y^t, W^n). \quad (4.24)$$

*s.t.*

$$(o^t, y^t) \in S^t. \quad (4.25)$$

Note that for a particular realization  $W^1, \dots, W^N$  of daily transportation requests,

the problem (4.24) - (4.25) is deterministic. We will refer to this corresponding deterministic problem as the SAA problem for *SPP*.

In order to solve the multi-period problem in which the decisions for all time periods are made simultaneously, we need to generate  $N$  realizations of travel demand for the entire planning horizon. Note that travel demand in the multi-period model can be represented by a random vector with each component denoting a set of transportation requests for a typical day in the corresponding time period. The SAA problem corresponding to *MPP* for the basic approach is

$$\min \widehat{z}_N = \sum_{t \in \mathcal{T}} \alpha^{t-1} \left( \frac{1}{N} \sum_{n=1}^N C(o^t, y^t, W^n) \right) \quad (4.26)$$

*s.t.*

$$(o^t, y^t) \in S^t. \quad \forall t \in \mathcal{T}. \quad (4.27)$$

Generic SAA methods work as follows (Santoso et al. [37], Kleywegt et al. [22]):

- Determine the number of SAA iterations  $M$ .  $M$  is generally set to be 10 or 20 in the literature.
- For  $m = 1, \dots, M$ ,
  - Generate  $N$  realizations of  $\mathbf{W}$ , i.e.  $(W_m^1, \dots, W_m^N)$ , and solve the corresponding SAA problem

$$\min_{x \in X} \left\{ \frac{1}{N} \sum_{n=1}^N G(x, W_m^n) \right\}.$$

Let  $\widehat{x}_N^m$  and  $z_N^m$  denote the corresponding optimal solution and optimal objective value, respectively.

- Compute



$$\bar{z}_N = \frac{1}{M} \sum_{m=1}^M z_N^m.$$

It is well known that  $\mathbb{E}[\bar{z}_N] \leq z^*$  where  $z^*$  represents the optimal value of the true problem (Mak et al. [24], Norkin et al. [30]). Thus,  $\bar{z}_N$  provides a statistical estimate for a lower bound on  $z^*$ . The variance of this estimator can be estimated by

$$\sigma_{\bar{z}_N}^2 = \frac{1}{(M-1)M} \sum_{m=1}^M (z_N^m - \bar{z}_N)^2.$$

- For any feasible solution  $\hat{x} \in X$ ,  $\mathbb{E}[G(\hat{x}, W)]$  is an upper bound for  $z^*$ . This upper bound can be estimated as follows. Generate  $N'$  realizations of  $\mathbf{W}$ , i.e.  $W^1, \dots, W^{N'}$ , that are independent of the realizations used to obtain  $\hat{x}_N^m$  for  $m = 1, \dots, M$ . Typically, one can take  $N'$  to be much bigger than  $N$  which is used in solving the SAA problems. Then, the estimator is

$$\hat{z}_{N'}(\hat{x}) = \frac{1}{N'} \sum_{n=1}^{N'} G(\hat{x}, W^n).$$

The variance of this estimator can be given as

$$\sigma_{\hat{z}_{N'}(\hat{x})}^2 = \frac{1}{(N'-1)N'} \sum_{n=1}^{N'} (G(\hat{x}, W^n) - \hat{z}_{N'}(\hat{x}))^2.$$

- Take  $\hat{x}^*$  to be the solution with the smallest estimated objective value among the optimal solutions  $\hat{x}_N^m$ , for  $m = 1, \dots, M$ , i.e.  $\hat{x}^* \in \arg \min\{\hat{z}_{N'}(\hat{x}) | \hat{x}_N^m \in \{\hat{x}_N^1, \dots, \hat{x}_N^M\}\}$ . Evaluate the quality of the solution by computing the optimality gap estimate  $\hat{z}_{N'}(\hat{x}^*) - \bar{z}_N$ . The estimated variance of this gap estimator is then given by  $\sigma_{\hat{z}_{N'}(\hat{x}^*)}^2 + \sigma_{\bar{z}_N}^2$ .
- If the estimate for the optimality gap, i.e.  $\hat{z}_{N'}(\hat{x}^*) - \bar{z}_N$ , is sufficiently small, then stop. Otherwise, increase  $N$  and continue.

The SAA algorithm used to solve  $SPP^t$  ( $MPP$ ) is given by Algorithm 4.2.

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**Algorithm 4.2** SAA ALGORITHM FOR SOLVING  $SPP^t$  ( $MPP$ )

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- 1: Determine the number of SAA iterations  $M$ , e.g.  $M = 10$  or  $M = 20$ .
  - 2: **for**  $m = 1, \dots, M$  **do**
  - 3:     Generate  $N$  realizations of daily transportation requests for time period  $t$  (for the entire planning horizon).
  - 4:     Solve the corresponding SAA problem using CPLEX and obtain the base location and fleet allocation decisions.
  - 5: **end for**
  - 6: Generate  $N'$  realizations of transportation requests independent of the ones used to solve the SAA problem where  $N' \gg N$  and compute the estimated objective values corresponding to the optimal solutions for each  $m$ .
  - 7: Choose the base location and fleet allocation decisions with the smallest estimated objective value, and compute the optimality gap estimate and its variance estimator for the chosen solution.
  - 8: **if** the estimate for the optimality gap of the chosen solution is sufficiently small **then**
  - 9:     Stop.
  - 10: **else**
  - 11:     Increase  $N$  and go to Step 2.
  - 12: **end if**
- 

### 4.3.2 Computational Results

In this section, we report and discuss base location and fleet allocation decisions produced by the basic approach for the operations of a PSOD air transportation provider, DayJet Corporation. We consider a planning horizon of 2 years during which 20 new jets are introduced every 3 months, i.e. base location and fleet allocation decisions are made quarterly. We start with 2 bases at the beginning of the planning horizon (one of them is where the headquarters of the business is located and the other one is the airport with the maintenance facility) and at each decision point one new base is opened. The business provides air transportation service between 44 airports located in the southeast US. The minimum and the maximum number of jets to be allocated to a base are 6 and 25, respectively. While finding the imbalances between the incoming and outgoing requests in the imbalance-based demand scheme,

it is assumed that all requests leave their origins at the middle of their departure time intervals and reach their destinations at the middle of their arrival time intervals.

The number of iterations for the SAA algorithm,  $M$ , is set to be 20 consistent with previous studies in the literature. Statistical validation of a candidate solution is carried out by evaluating the objective function using  $N' = 1000$  realizations of daily transportation requests. We first present the results where decisions are made period by period during the planning horizon. Table 4.1 and Table 4.2 summarize the optimality gap estimates of the best solutions for each time period obtained with different number of daily transportation request realizations,  $N$ , for the activity-based and the imbalance-based demand schemes, respectively. The third column ( $\hat{z}_{N'}(\hat{x}^*)$ ) corresponds to the objective value estimate of the best solution. The fourth column ( $gap^{abs}$ ) denotes the absolute optimality gap estimate for  $\hat{z}_{N'}(\hat{x}^*)$  which is calculated as  $\hat{z}_{N'}(\hat{x}^*) - \bar{z}_N$ . The fifth column ( $\sigma_{gap^{abs}}$ ) represents the estimate for the standard deviation of the the absolute optimality gap estimate. Finally, the sixth column (gap) denotes the percentage gap of the estimated objective values from the lower bounds. The results indicate that high-quality solutions can be obtained using the SAA method even with such small numbers of travel demand realizations since all solutions are within an estimated 1% from the lower bounds. Furthermore, as can be seen from the tables,  $N = 30$  is sufficient to obtain solutions with very small optimality gap estimates. Thus, from this point onwards the best solutions obtained with  $N = 30$  will be used for discussion.

Table 4.3 and Table 4.4 show the base location and fleet allocation decisions made at each time period for the activity-based and the imbalance-based demand schemes, respectively. As more bases are opened, the number of jets allocated to each base becomes more evenly distributed for both demand schemes. Figure 4.4 and Figure 4.5 show the airports that are opened as bases with the activity-based and the imbalance-based demand schemes, respectively. The airports represented with

**Table 4.1:** Optimality gap estimates for the activity-based demand scheme

Period 1	$N$	$\widehat{z}_{N'}(\widehat{x}^*)$	$gap^{abs}$	$\sigma_{gap^{abs}}$	gap
	10	1306.48	7.81	0.46	0.6
	20	1306.48	2.02	0.35	0.15
	30	1306.48	0.89	0.09	0.07
	40	1306.48	0.39	0.07	0.03
	50	1306.48	0.28	0.06	0.02
Period 2	$N$	$\widehat{z}_{N'}(\widehat{x}^*)$	$gap^{abs}$	$\sigma_{gap^{abs}}$	gap
	10	1780.12	5.32	0.69	0.3
	20	1780.12	1.4	0.47	0.08
	30	1780.12	1.37	0.12	0.08
	40	1780.12	1.05	0.09	0.06
	50	1780.12	0.22	0.05	0.01
Period 3	$N$	$\widehat{z}_{N'}(\widehat{x}^*)$	$gap^{abs}$	$\sigma_{gap^{abs}}$	gap
	10	2340.36	6.15	0.21	0.26
	20	2340.36	4.06	0.11	0.17
	30	2340.36	3.11	0.09	0.13
	40	2340.36	1.96	0.08	0.08
	50	2340.36	0.3	0.03	0.01
Period 4	$N$	$\widehat{z}_{N'}(\widehat{x}^*)$	$gap^{abs}$	$\sigma_{gap^{abs}}$	gap
	10	2696.76	8.02	0.61	0.3
	20	2696.76	3.38	0.42	0.13
	30	2696.76	1.46	0.08	0.05
	40	2696.76	0.35	0.08	0.01
	50	2696.76	0.09	0.03	0.003
Period 5	$N$	$\widehat{z}_{N'}(\widehat{x}^*)$	$gap^{abs}$	$\sigma_{gap^{abs}}$	gap
	10	3089.15	7.85	0.62	0.25
	20	3089.15	3.12	0.52	0.1
	30	3089.15	1.91	0.07	0.06
	40	3089.15	1.82	0.06	0.06
	50	3089.15	0.3	0.06	0.01
Period 6	$N$	$\widehat{z}_{N'}(\widehat{x}^*)$	$gap^{abs}$	$\sigma_{gap^{abs}}$	gap
	10	3431.69	11.3	0.43	0.33
	20	3431.69	5.51	0.35	0.16
	30	3431.69	2.12	0.03	0.06
	40	3431.69	1.04	0.03	0.03
	50	3431.69	0.74	0.03	0.02
Period 7	$N$	$\widehat{z}_{N'}(\widehat{x}^*)$	$gap^{abs}$	$\sigma_{gap^{abs}}$	gap
	10	3763.54	8.14	0.3	0.22
	20	3763.54	3.69	0.27	0.1
	30	3763.36	2.32	0.07	0.06
	40	3763.36	1.37	0.05	0.04
	50	3763.36	1.03	0.04	0.03
Period 8	$N$	$\widehat{z}_{N'}(\widehat{x}^*)$	$gap^{abs}$	$\sigma_{gap^{abs}}$	gap
	10	4051.23	12.39	0.5	0.31
	20	4050.8	3.75	0.43	0.09
	30	4050.8	0.76	0.09	0.02
	40	4050.8	0.2	0.09	0.005
	50	4050.8	0.06	0.04	0.001
Period 9	$N$	$\widehat{z}_{N'}(\widehat{x}^*)$	$gap^{abs}$	$\sigma_{gap^{abs}}$	gap
	10	4306.06	19.1	0.93	0.44
	20	4303.85	10.48	0.39	0.24
	30	4303.85	5.69	0.06	0.13
	40	4303.85	4.56	0.06	0.1
	50	4303.85	4.21	0.04	0.1

**Table 4.2:** Optimality gap estimates for the imbalance-based demand scheme

Period 1	$N$	$\widehat{z}_{N'}(\widehat{x}^*)$	$gap^{abs}$	$\sigma_{gap^{abs}}$	gap
	10	1157.66	6.11	0.61	0.53
	20	1157.66	4.14	0.44	0.36
	30	1157.66	3.21	0.06	0.28
	40	1157.66	2.84	0.05	0.25
	50	1157.66	0.42	0.03	0.04
Period 2	$N$	$\widehat{z}_{N'}(\widehat{x}^*)$	$gap^{abs}$	$\sigma_{gap^{abs}}$	gap
	10	1680.17	6.93	0.46	0.41
	20	1680.17	2.05	0.36	0.12
	30	1680.17	0.7	0.03	0.04
	40	1680.17	0.54	0.03	0.03
	50	1680.17	0.45	0.02	0.02
Period 3	$N$	$\widehat{z}_{N'}(\widehat{x}^*)$	$gap^{abs}$	$\sigma_{gap^{abs}}$	gap
	10	2201.41	8.61	0.57	0.39
	20	2201.41	3.95	0.48	0.18
	30	2201.41	2.82	0.06	0.12
	40	2201.41	1.84	0.08	0.08
	50	2340.36	1.33	0.05	0.06
Period 4	$N$	$\widehat{z}_{N'}(\widehat{x}^*)$	$gap^{abs}$	$\sigma_{gap^{abs}}$	gap
	10	2685.97	10.59	0.78	0.39
	20	2685.92	9.02	0.34	0.34
	30	2685.79	5.58	0.05	0.21
	40	2685.79	4.94	0.05	0.19
	50	2685.64	3.1	0.06	0.12
Period 5	$N$	$\widehat{z}_{N'}(\widehat{x}^*)$	$gap^{abs}$	$\sigma_{gap^{abs}}$	gap
	10	3085.89	8.16	0.99	0.26
	20	3084.69	6.42	0.81	0.21
	30	3084.66	3.64	0.06	0.12
	40	3084.66	2.19	0.05	0.07
	50	3084.66	1.51	0.05	0.05
Period 6	$N$	$\widehat{z}_{N'}(\widehat{x}^*)$	$gap^{abs}$	$\sigma_{gap^{abs}}$	gap
	10	3420.72	5.56	0.36	0.16
	20	3420.72	4.46	0.29	0.13
	30	3420.42	1.74	0.09	0.05
	40	3420.25	0.62	0.06	0.02
	50	3420.25	0.52	0.04	0.02
Period 7	$N$	$\widehat{z}_{N'}(\widehat{x}^*)$	$gap^{abs}$	$\sigma_{gap^{abs}}$	gap
	10	3783.18	8.22	0.28	0.22
	20	3781.56	5.05	0.13	0.13
	30	3781.56	2.78	0.07	0.07
	40	3781.56	1.96	0.07	0.05
	50	3781.56	0.87	0.05	0.02
Period 8	$N$	$\widehat{z}_{N'}(\widehat{x}^*)$	$gap^{abs}$	$\sigma_{gap^{abs}}$	gap
	10	4053.16	11	0.87	0.27
	20	4052.84	3	0.51	0.07
	30	4052.17	2.37	0.12	0.06
	40	4052.17	1.64	0.1	0.04
	50	4052.17	1.02	0.1	0.03
Period 9	$N$	$\widehat{z}_{N'}(\widehat{x}^*)$	$gap^{abs}$	$\sigma_{gap^{abs}}$	gap
	10	4298.73	13.8	0.91	0.32
	20	4298.73	9.34	0.61	0.22
	30	4297.45	4.38	0.07	0.1
	40	4296.16	3.67	0.06	0.09
	50	4296.16	2.89	0.06	0.07

the circles are opened as bases whereas the airports with the squares are not. The sizes of the circles in the figures are proportional to the number of jets allocated to corresponding bases at the end of the planning horizon. Furthermore, the number next to a base corresponds to the period in which it is opened. As can be seen from the figures, the period in which a base with a certain location relative to all other airports is opened differs slightly between the solutions. For example, in the activity-based demand scheme a base is located in the north-east corner of region in period 2 whereas a base is not located in a location close to that corner until time period 6 in the imbalance-based demand scheme.

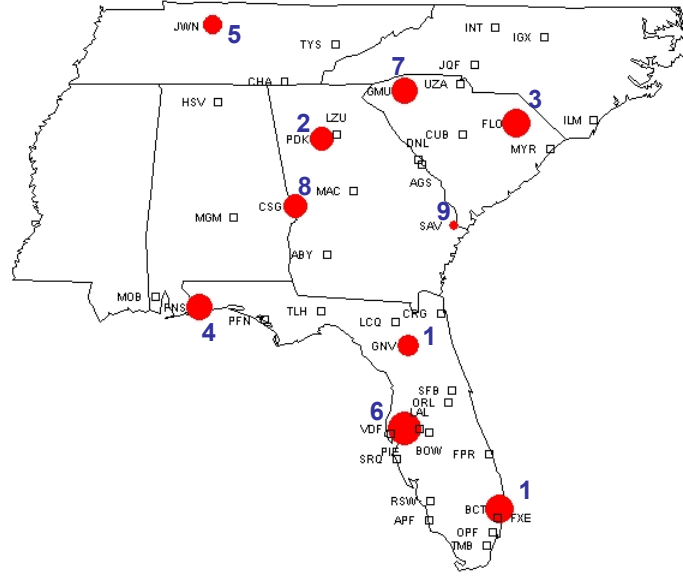
**Table 4.3:** Base location and fleet allocation decisions for each time period for the activity-based demand scheme

Period	Base Location and Fleet Allocation Decisions
1	BCT(6) GNV(14)
2	BCT(6) GNV(11) PDK(23)
3	BCT(9) GNV(16) PDK(24) FLO(11)
4	BCT(12) GNV(17) PDK(25) FLO(15) PNS(11)
5	BCT(15) GNV(21) PDK(23) FLO(19) PNS(14) JWN(8)
6	BCT(14) GNV(14) PDK(25) FLO(23) PNS(12) JWN(11) VDF(16)
7	BCT(17) GNV(16) PDK(22) FLO(20) PNS(19) JWN(11) VDF(18) GMU(17)
8	BCT(19) GNV(17) PDK(14) FLO(22) PNS(18) JWN(13) VDF(21) GMU(20) CSG(16)
9	BCT(22) GNV(16) PDK(19) FLO(22) PNS(20) JWN(14) VDF(23) GMU(19) CSG(17) SAV(8)

**Table 4.4:** Base location and fleet allocation decisions for each time period for the imbalance-based demand scheme

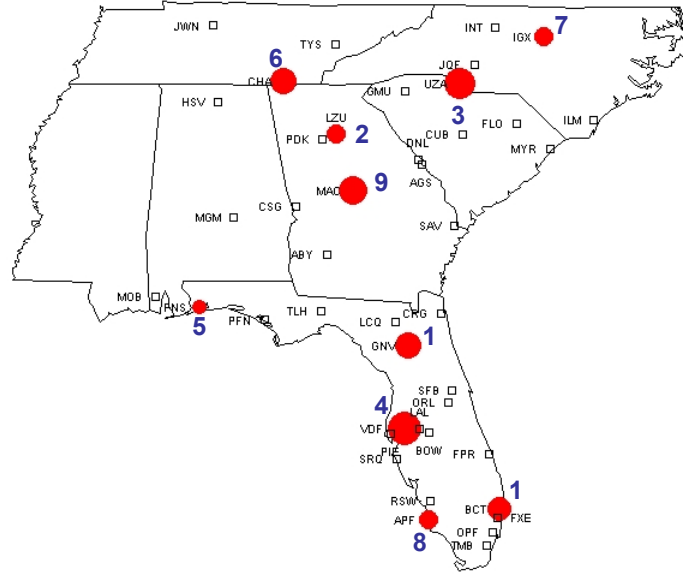
Period	Base Location and Fleet Allocation Decisions
1	BCT(6) GNV(14)
2	BCT(8) GNV(11) LZU(21)
3	BCT(12) GNV(16) LZU(19) UZA(11)
4	BCT(12) GNV(13) LZU(25) UZA(17) VDF(13)
5	BCT(15) GNV(13) LZU(23) UZA(23) VDF(16) PNS(8)
6	BCT(19) GNV(16) LZU(20) UZA(25) VDF(18) PNS(9) CHA(13)
7	BCT(22) GNV(17) LZU(23) UZA(20) VDF(22) PNS(10) CHA(15) IGX(11)
8	BCT(15) GNV(20) LZU(25) UZA(24) VDF(22) PNS(11) CHA(18) IGX(12) APF(13)
9	BCT(17) GNV(20) LZU(15) UZA(23) VDF(24) PNS(11) CHA(20) IGX(14) APF(15) MAC(21)

The quality of a set of base location and fleet allocation decisions is measured



**Figure 4.4:** Bases opened with the activity-based demand scheme

by using these decisions in the operational level flight scheduling. Specifically, we consider the acceptance rate for the transportation requests and the total daily flying time over all the jets obtained with the corresponding base location and fleet allocation decisions. In order to assess the quality, we determine which transportation requests can be accommodated and construct itineraries for the requests that can be accommodated for 30 days within each time period using the algorithms and decision support tools developed by DayJet for online and off-line flight scheduling. The base location and fleet allocation decisions made for each time period are inputs to these algorithms. Table 4.5 shows the average over 30 days of the acceptance rate measured by the percentage of transportation requests that can be accommodated, for both demand schemes. As can be seen from the table, the activity-based demand scheme has an average acceptance rate of 96.9% whereas the imbalance-based demand scheme has an average acceptance rate of 95.8%. This is expected since the activity-based demand scheme tries to give more weight to airports with more outgoing requests



**Figure 4.5:** Bases opened with the imbalance-based demand scheme

during the day in order to be able to accommodate more transportation requests.

Given a set of transportation requests to be accommodated on a day, the itineraries are constructed next. The total daily flying time is measured as the total flying time (in minutes) accrued by all the jets in the fleet in order to satisfy the accepted transportation requests. Thus, in order to have a fair comparison between the average daily flying times obtained by using different demand schemes, any transportation requests that cannot be accommodated by one of the approaches is removed from consideration while constructing the itineraries. Table 4.6 shows the average over 30 days of the total daily flying times for both demand schemes given the same set of transportation requests. The last column in the table represented by **(Ratio)** measures the ratio of the average daily flying time obtained with the imbalance-demand scheme to the one obtained with the activity-based demand scheme. The base location and fleet allocation decisions made with the imbalance-based demand scheme result in less average daily flying time compared to the decisions made using



**Table 4.5:** Average acceptance rate for the transportation requests with the basic approach

Time period	Activity-based	Imbalance-based
1	99.8	99.5
2	98.6	97.3
3	97.3	96.8
4	97.7	96.2
5	97.6	96.3
6	96.2	95.4
7	95.7	94.2
8	94.9	93.6
9	94.2	93.2
Average	96.9	95.8

**Table 4.6:** Average of the total daily flying time with the basic approach

Time period	Activity-based	Imbalance-based	Ratio
1	6659.3	6612.8	0.99
2	14887.3	14843.6	0.99
3	32272.7	31865.5	0.98
4	40869.1	40718.2	0.99
5	50121.4	48248.2	0.96
6	65045.5	64423.4	0.99
7	73711.9	73109.2	0.99
8	85625.7	85463.7	0.99
9	98374.1	93350.5	0.95
Average	51951.9	50959.4	0.985

the activity-based demand scheme. Specifically, at each time period the average daily flying time for the imbalance-based demand scheme is around 98.5% of the average daily flying time for the activity-based demand scheme. Thus, the average daily flying time can be decreased by on average 1.5% by more carefully considering the impact of the imbalances between the incoming and outgoing requests at an airport to avoid deadheads.

In order to analyze the quality of the base location and fleet allocation decisions made period by period during the planning horizon, we next report the decisions

and their corresponding outcomes in the operational flight scheduling for the perfect information case where the decisions for all time periods are made simultaneously. In order to calculate the present value of the future costs, we use a quarterly interest rate of 1%. The base location and fleet allocation decisions for each time period made with the activity-based and imbalance-based demand schemes are given in Table 4.7 and Table 4.8, respectively. The number of jets allocated to each base at each time period is more evenly distributed when decisions for all time periods are made simultaneously.

**Table 4.7:** Base location and fleet allocation decisions for each time period for the activity-based demand scheme when all decisions are made simultaneously

Period	Base Location and Fleet Allocation Decisions
1	BCT(6) GNV(14)
2	BCT(12) GNV(15) LAL(13)
3	BCT(11) GNV(18) LAL(19) TLH(12)
4	BCT(16) GNV(20) LAL(15) TLH(21) FLO(8)
5	BCT(11) GNV(22) LAL(18) TLH(13) FLO(16) HSV(20)
6	BCT(18) GNV(20) LAL(16) TLH(15) FLO(18) HSV(18) LZU(15)
7	BCT(17) GNV(21) LAL(17) TLH(16) FLO(19) HSV(16) LZU(16) UZA(18)
8	BCT(16) GNV(19) LAL(19) TLH(18) FLO(15) HSV(17) LZU(18) UZA(19) AGS(19)
9	BCT(20) GNV(18) LAL(16) TLH(21) FLO(17) HSV(18) LZU(22) UZA(19) AGS(18) ABY(11)

**Table 4.8:** Base location and fleet allocation decisions for each time period for the imbalance-based demand scheme when all decisions are made simultaneously

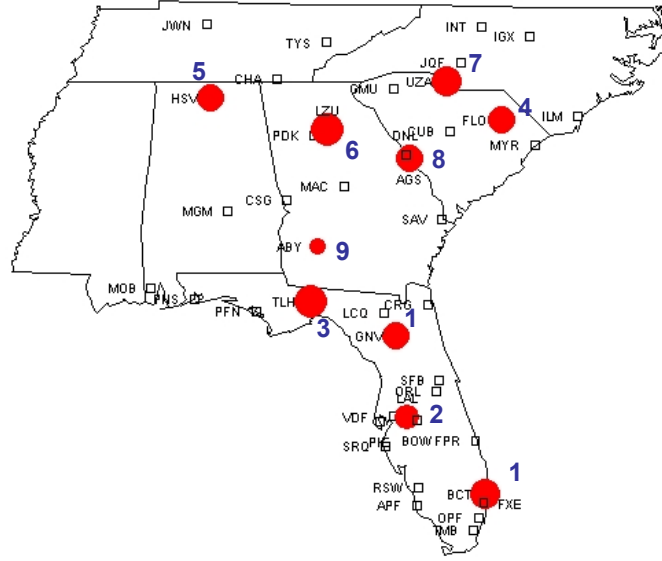
Period	Base Location and Fleet Allocation Decisions
1	BCT(6) GNV(14)
2	BCT(9) GNV(17) UZA(14)
3	BCT(11) GNV(16) UZA(18) PDK(15)
4	BCT(12) GNV(15) UZA(21) PDK(16) LAL(16)
5	BCT(14) GNV(13) UZA(22) PDK(18) LAL(14) TLH(19)
6	BCT(17) GNV(16) UZA(25) PDK(16) LAL(15) TLH(21) CHA(10)
7	BCT(17) GNV(18) UZA(22) PDK(20) LAL(16) TLH(19) CHA(18) FLO(10)
8	BCT(19) GNV(17) UZA(18) PDK(20) LAL(18) TLH(17) CHA(19) FLO(15) APF(17)
9	BCT(20) GNV(16) UZA(19) PDK(21) LAL(20) TLH(16) CHA(21) FLO(17) APF(18) ABY(12)

Figures 4.6 and 4.7 show the base location and fleet allocation decisions on the map, respectively. The relative locations of the bases opened at each time period

**Table 4.9:** Summary statistics for flight scheduling with the basic approach when decisions for all time periods are made simultaneously

Time period	Activity-based		Imbalance-based	
	Avg. Acc. Rate	Avg. daily flying time	Avg. Acc. Rate	Avg. daily flying time
1	99.8	6592.7	99.7	6566.5
2	99.1	14738.4	97.9	14739.7
3	97.6	31788.6	96.3	31642.4
4	97.5	40460.4	97.1	40433.2
5	97.7	49620.2	97.4	47910.5
6	97.1	64395.1	96.4	63972.4
7	96.2	72974.8	95.8	72963
8	95.1	84769.8	94.6	84865.5
9	94.8	97390.4	93.9	92743.7
Average	97.2	51414.5	96.6	50648. 5

is very similar to the ones that are determined period by period for both demand schemes. The summary statistics for the operational flight scheduling obtained with the base location and fleet allocation decisions that are made simultaneously for all time periods are given in Table 4.9. The summary statistics in Table 4.10 show that when the decisions are made simultaneously, the acceptance rate for the transportation requests increases by on average 0.5% (0.3 % and 0.8 % for the activity-based and imbalance-based demand schemes, respectively), and the average daily flying time decreases by on average 0.8% (1 % and 0.6 % for the activity-based and imbalance-based demand schemes, respectively). Thus, we conclude that when making the base location and fleet allocations period by period, we can make decisions almost as good as the ones that would have been made if all decisions were made simultaneously at the beginning of the planning horizon with perfect information. In practice travel demand forecasts are updated quarterly and thus, making all decisions at the beginning of the planning horizon without the necessary information for future demand is not relevant.



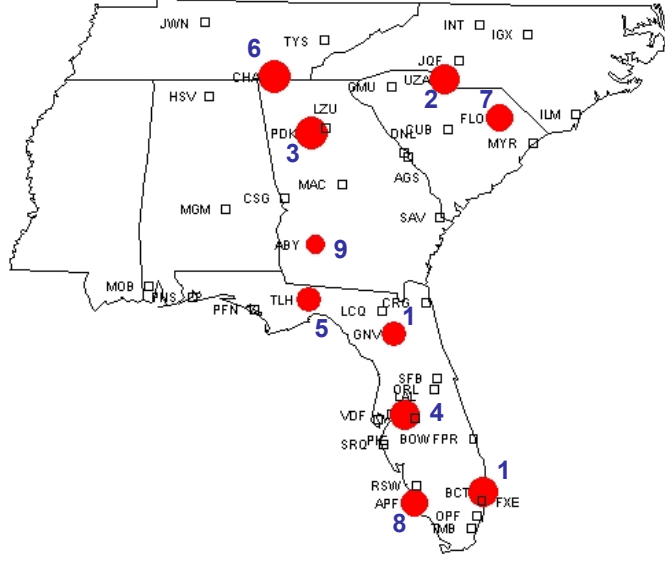
**Figure 4.6:** Bases opened with the activity-based demand scheme when decisions for all time periods made simultaneously

#### 4.4 Integrated Approach

The only way flight scheduling is captured in the basic approach is through aggregation of transportation requests which is a very simple approximation of flight scheduling. The goal of the integrated approach is to incorporate flight scheduling at a more detailed level. Thus, we develop a model that works directly with the transportation requests and integrates a simplified version of flight scheduling with the base location

**Table 4.10:** Summary of the average acceptance rate and average daily flying time for the basic approach

	Avg. Acc. Rate	Avg. daily flying time
Activity-based (Period by period)	96.9	51951.9
Activity-based (Perfect information)	97.2	51414.5
Imbalance-based (Period by period)	95.8	50959.4
Imbalance-based (Perfect information)	96.6	50648.5



**Figure 4.7:** Bases opened with the imbalance-based demand scheme when decisions for all time periods made simultaneously

and fleet allocation decisions. That is, the base location and fleet allocation decisions are made while determining the decisions regarding the routing of the transportation requests and the jets.

For a given set of daily transportation requests in time period  $t$ , we construct a time-expanded network denoted by  $\mathcal{D} = (\mathcal{N}, \mathcal{E})$  where  $\mathcal{N}$  is the set of nodes and  $\mathcal{E}$  is the set of arcs. We discretize the time during the day into regular intervals. The nodes in  $\mathcal{N}$  represent the airports at different time points during the day. Let the set of time points during the day be denoted by  $\mathcal{U}$ . There are two types of arcs in  $\mathcal{E}$ . The first type of arcs are called the *travel arcs* and the set of these arcs is represented by  $\mathcal{E}^D$ . An arc  $e = (n_1, n_2) \in \mathcal{E}^D$  wherever  $n_1 = (a, u_1)$  and  $n_2 = (b, u_2)$  is such that  $u_2 \geq u_1 + t_{ab}^f$ , i.e the time difference between  $u_2$  and  $u_1$  is larger than the direct travel time from airport  $a$  to airport  $b$ . The second type of arcs are called the ground arcs and the set of these arcs is represented by  $\mathcal{E}^G$ . An arc  $e = (n_1, n_2) \in \mathcal{E}^G$  wherever

$n_1 = (a, u_1)$  and  $n_2 = (b, u_2)$  is such that  $a = b$  and  $u_2$  is the next time point after  $u_1$ . The flying time on arc  $e$  is denoted by  $t_e^f$  and is equal to the flying time from airport  $a$  to airport  $b$  which is  $t_{ab}^f$ .

We next construct feasible paths in the network to route each transportation request from its origin to its destination. The set of feasible paths for request  $r \in \mathcal{R}$  is denoted by  $\mathcal{P}_r$ . As discussed earlier, the maximum number of allowed intermediate stops on path  $p \in \mathcal{P}_r$  is usually small for a PSOD air transportation service. Consider the example where at most one intermediate stop is allowed. Then, a path  $p \in \mathcal{P}_r$  consists of either one arc that corresponds to the direct flight from the origin of request  $r$  to the destination of request  $r$  or two arcs where the first arc corresponds to the flight from the origin of request  $r$  to an intermediate stop and the second arc corresponds to the flight from the intermediate stop to the destination of request  $r$ .

The flight scheduling problem for PSOD air transportation is a computationally NP-hard problem (Espinoza et al. [12, 13] and Engineer et al. [11]). Thus, we need to make certain simplifying assumptions to make our problem that combines these decisions with the tactical level base location and fleet allocation decisions tractable. The first assumption concerns the time discretization. We use a coarser time discretization compared to the original flight scheduling problem, for example half an hour discretization versus one minute discretization of flight scheduling. The second simplification is in the construction of the feasible paths to route the transportation requests. In our integrated model, all feasible paths to route a request are not constructed since the number of such paths is quite large. Instead, while constructing the feasible paths it is assumed that a request has to leave its origin or its intermediate stop at one of the earliest  $k$  time points it is allowed to do so. For example, if  $k = 3$ , for a transportation request with an earliest departure time of 7 am, we consider 7 am, 7:30 am and 8 am as possible departure times if the destination can be reached before the latest arrival time.

There are four types of decision variables in the formulation. The binary variable  $o_a^t$  and the integer variable  $y_a^t$  are the same as their counterparts in the basic approach. The integer variable  $x_e$  denotes the amount of flow for jets on arc  $e \in \mathcal{E}$ . Finally, the fourth type of decision variable is binary variable  $f_r^p$  which equals 1 if path  $p \in \mathcal{P}_r$  is used to satisfy transportation request  $r \in \mathcal{R}$ . Let  $c$  denote the seating capacity of a jet and  $r^n$  denote the number of passengers associated with transportation request  $r$ . Let  $u^0$  and  $u^{|\mathcal{U}|}$  denote the first and the last time points during the day, respectively. The deterministic single period problem at time period  $t$  for a particular set of transportation requests  $\mathcal{R}$  is

$$(SPP^{\mathcal{R}}) : \min \sum_{e \in \mathcal{E}} t_e^f x_e \quad (4.28)$$

s.t.

$$(o^t, y^t) \in S^t \quad (4.29)$$

$$\sum_{e \in \delta^{out}(n)} x_e - \sum_{e \in \delta^{in}(n)} x_e = \begin{cases} y_a^t & \text{if } n = (a, u^0); \\ -y_a^t & \text{if } n = (a, u^{|\mathcal{U}|}); \\ 0 & \text{otherwise.} \end{cases} \quad \forall n \in \mathcal{N} \quad (4.30)$$

$$\sum_{p \in \mathcal{P}_r} f_r^p = 1 \quad \forall r \in \mathcal{R} \quad (4.31)$$

$$\sum_{p \in \mathcal{P}_r : e \in p} r^n f_r^p \leq c x_e \quad \forall e \in \mathcal{E} \quad (4.32)$$

$$x_e \geq 0 \quad \forall e \in \mathcal{E} \quad (4.33)$$

$$f_r^p \in \{0, 1\} \quad \forall r \in \mathcal{R}, p \in \mathcal{P}_r. \quad (4.34)$$

The objective is to minimize the operational cost which is approximated by the total flying time. Constraints (4.29) are the common set of constraints regarding the base location and fleet allocation decisions with the basic approach. Constraints (4.30) are the flow balance constraints for the jets. Constraints (4.31) ensure that

all requests are routed from their origins to their destinations. Finally and most importantly, constraints (4.32) are the seating capacity constraints for the jets that link the flow of requests to the flow of jets in the network. These constraints have the most significant effect on the decisions as they determine the flow of jets necessary to satisfy the requirements of the transportation requests.

#### 4.4.1 Solution Approach

Given the amount of detail about flight scheduling captured in the integrated approach, the multi-period problem in which the base location and fleet allocation decisions are made simultaneously for all time periods becomes intractable even with the simplifying assumptions made regarding flight scheduling. Thus, we make the base location and fleet allocation decisions only period by period while using the integrated approach.

Consider the single period problem at time period  $t$ . The transportation requests for a day in time period  $t$  are not known with certainty. In the basic approach, we addressed the uncertainty regarding the transportation requests explicitly by solving a two-stage stochastic problem. However, given the complexity of the model used in the integrated approach, we cannot incorporate the uncertainty into the model. Instead, in order to take the variability in the set of transportation requests  $\mathcal{R}$  into account, we solve  $SPP^{\mathcal{R}}$  with several different realizations of  $\mathcal{R}$ . The base location and fleet allocation decisions made for each realization of  $\mathcal{R}$  are then merged into one set of decisions using a two-step optimization method. While obtaining a single set of base location and fleet allocation decisions, the aim is to end up with the decisions that are as close to the individual solutions as possible.

Let  $(o_{\mathcal{R}}^t, y_{\mathcal{R}}^t)$  be the base location and fleet allocation decisions made by solving  $SPP^{\mathcal{R}}$ . In the first step, we determine the airports that are operating as bases by solving the problem



$$\min z = \sum_{a \in \mathcal{A}} \sum_{r \in \mathcal{R}} (o_a^t - o_{\mathcal{R}}^t)^2 \quad (4.35)$$

*s.t.*

$$\sum_{a \in \mathcal{A}} o_a^t = n_t^B. \quad (4.36)$$

After determining the airports that will be operating as bases, the number of jets to be allocated to each of these airports is found in the second step by solving the problem

$$\min z = \sum_{a \in \mathcal{A}} \sum_{r \in \mathcal{R}} (y_a^t - y_{\mathcal{R},a}^t)^2 \quad (4.37)$$

*s.t.*

$$y_a^t \geq n_{min}^j \bar{o}_a^t \quad \forall a \in \mathcal{A} \quad (4.38)$$

$$y_a^t \leq n_{max}^j \bar{o}_a^t \quad \forall a \in \mathcal{A} \quad (4.39)$$

$$\sum_{a \in \mathcal{A}} y_a^t = |\mathcal{J}_t| \quad (4.40)$$

where  $\bar{o}_a^t$  equals 1 if a base is chosen to be operating at airport  $a$  and 0 otherwise.

#### 4.4.2 Computational Results

In this section, we report the results obtained with the integrated approach for solving the base location and fleet allocation problem of DayJet Corporation. At each time period, we solve  $SPP^{\mathcal{R}}$  for 30 different realizations of  $\mathcal{R}$ . That is, we obtain 30 solutions by solving the single period deterministic problem in which the transportation requests and the jets are routed while the base location and fleet allocation decisions are made by using 30 different sets of daily transportation requests. Then, we merge these individual solutions to a single solution using the two-step optimization method. We discretize the time into 30 minute intervals and we set  $k = 3$  (the parameter that

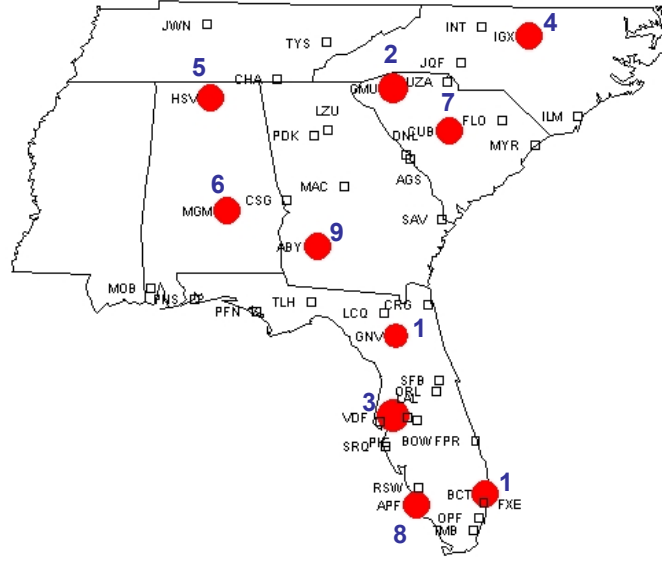
**Table 4.11:** Base location and fleet allocation decisions for each time period made with the integrated approach

Period	Base Location and Fleet Allocation Decisions
1	BCT(6) GNV(14)
2	BCT(12) GNV(14) GMU(14)
3	BCT(13) GNV(16) GMU(15) VDF(16)
4	BCT(14) GNV(17) GMU(16) VDF(17) IGX(16)
5	BCT(17) GNV(16) GMU(18) VDF(16) IGX(19) HSV(14)
6	BCT(16) GNV(18) GMU(17) VDF(18) IGX(17) HSV(20) MGM(14)
7	BCT(18) GNV(16) GMU(19) VDF(20) IGX(18) HSV(19) MGM(17) CUB(13)
8	BCT(19) GNV(17) GMU(18) VDF(18) IGX(19) HSV(17) MGM(19) CUB(16) APF(17)
9	BCT(18) GNV(16) GMU(19) VDF(21) IGX(18) HSV(17) MGM(18) CUB(17) APF(18) ABY(18)

determines how many time points a request can leave a certain airport). Finally, the seating capacity of all the jets is the same and equal to 3.

Table 4.11 gives the base location and fleet allocation decisions made for each time period with the integrated approach. The number of jets allocated to each base becomes more evenly distributed as more bases are opened for both the basic and the integrated approaches. However, the results in Table 4.11 show that the number of jets allocated to each base is evenly distributed at each time period for the integrated approach. Figure 4.8 shows the airports that are opened as bases with the integrated approach together with the time period in which they are opened. We observe that the decisions made with the basic approach using the imbalance-based demand scheme are more similar to the ones made with the integrated approach than the ones made with the basic approach using the activity-based demand scheme. Thus, we can conclude that just by considering the imbalances between the incoming and outgoing requests, the imbalance-based demand scheme captures some valuable information about flight scheduling which the activity-based demand scheme cannot.

Table 4.12 shows the summary statistics for the operational flight scheduling obtained by using the base location and fleet allocation decisions made by the integrated approach. The averages in each time period are obtained over the same 30 days used



**Figure 4.8:** Bases opened with the integrated approach

in the computational study for the basic approach. Table 4.13 summarizes the average acceptance rate and the average daily flying time statistics for the basic and the integrated approaches. When the average acceptance rate of 98.59% is compared to the best average acceptance rate obtained with the basic approach (the average acceptance rate obtained with the activity-based demand scheme), which is 96.9%, it is seen that an average of 2% improvement can be achieved with the integrated approach. Furthermore, the average daily flying time of 49083.96 minutes obtained with the integrated approach is around 4 % less than the best average daily flying time of 50959.4 minutes (average daily flying time obtained with the imbalance-based demand scheme) obtained with the basic approach. Thus, incorporating more detail about travel demand and flight scheduling into tactical decision making can result in accommodating more demand and it can also decrease the operational costs for satisfying a fixed set of transportation requests.

**Table 4.12:** Summary statistics for flight scheduling with the integrated approach

Time period	Avg. Acc. Rate	Avg. daily flying time
1	100	6330.43
2	99.6	14428.02
3	99.5	30654.56
4	99.1	39130.17
5	98.7	46849.01
6	98.4	61975.29
7	97.5	70842.79
8	97.8	82301.54
9	96.7	89243.87
Average	98.59	49083.96

**Table 4.13:** Summary of the average acceptance rate and average daily flying time for both approaches

	Avg. Acc. Rate	Avg. daily flying time
Activity-based (Period by period)	96.9	51951.9
Imbalance-based (Period by period)	95.8	50959.4
Integrated approach	98.59	49083.96

## CHAPTER V

### CONCLUSIONS AND FUTURE RESEARCH

We have addressed two important planning problems of per-seat, on-demand (PSOD) air transportation: scheduled maintenance planning, and base location and fleet allocation. For each problem, modeling and solution approaches have been developed for tactical and operational level decision making.

In Chapter 2, we have studied the tactical level maintenance capacity planning that determines the maximum number of jets that can be maintained on a day. Two operating conditions have been considered: a growth phase where jets are introduced gradually into the system and the steady state where the fleet size is constant. The optimization-based local search that has been developed to solve the tactical planning problem in the growth phase has been shown to obtain solutions within less than 1% of optimality in 2 hours for a fleet of jets that grows to a size of 480 in 2 years. Furthermore, it has been shown that by carefully planning the arrival date of the jets, total capacity required for maintenance can be decreased by up to 14%. Tactical planning for steady state maintenance capacity concerns the special case in which an equal number of jets are introduced on consecutive days at the beginning of the planning horizon. A pseudo-polynomial time algorithm has been given to determine the optimal and the long run capacities. A direction for future research is to generalize the results for any jet arrival schedule.

In Chapter 3, we have addressed operational planning for scheduled maintenance where the specific jets to be maintained on each day are determined. We have presented a solution methodology that can efficiently schedule maintenance of 480 jets

over a two year planning horizon together with their itinerary assignments. The decisions for a single day can be made on average within 12 seconds. We have also developed an integrated framework that captures the interaction between operational level maintenance decisions and flight scheduling. A simulated case study for the operations of a PSOD air transportation provider demonstrates that high-quality solutions to real-life instances can be achieved using this framework. An interesting aspect of scheduled maintenance to be studied in the future is the synchronization between different maintenance types, such as considering the maintenance that has to be done every 300 hours together with the maintenance that has to be done after every 100 take-offs and landings.

In Chapter 4, we have presented the tactical planning problem of base location and fleet allocation. We have investigated the impact of the level of detail regarding flight scheduling captured at the tactical level decision making on the operational level flight scheduling decisions. Specifically, we have considered two approaches to incorporate different levels of detail: a basic approach which captures high level information and an integrated approach which incorporates more detailed information by combining the base location and fleet allocation decisions with a simplified version of operational flight scheduling. It has been shown that when more flight scheduling detail is considered at the tactical level, the resulting base location and fleet allocation decisions lead to an average increase of 2% in the acceptance rate for the travel requests and for a given set of transportation requests to be accommodated, the average daily flying time can be decreased by on average 4%. Future research can consider the impact of fixed cost of opening bases on the base location and fleet allocation decisions, specifically with different levels of fixed cost for different sizes of bases to be opened.

Although motivated by the operations of PSOD air transportation, the models and solution approaches introduced in this thesis can be used more generally. For

example, the models and solution approaches for scheduled maintenance planning can be used for dynamic scheduling problems in which assets need to undergo periodic maintenance and the arrival and the length of the tasks that need to be performed by these assets are non-deterministic. Furthermore, the models and solution approaches developed for base location and fleet allocation problem can be used for any tactical location problem that has a routing component at the operational level, for example determining the locations of depots where at the operational level the daily routes for the vehicles are constructed.

## REFERENCES

- [1] AGARWAL, R. and ERGUN, O., “Ship scheduling and network design for cargo routing in liner shipping,” *Transportation Science*, vol. 42, pp. 175–196, 2008.
- [2] ANILY, S., GLASS, C. A., and HASSIN, R., “The scheduling of maintenance service,” *Discrete Applied Mathematics*, vol. 82, pp. 27–42, 1998.
- [3] ANILY, S., GLASS, C. A., and HASSIN, R., “Scheduling maintenance services to three machines,” *Annals of Operations Research*, vol. 86, pp. 375–391, 1999.
- [4] BAR-NOY, A., BHATIA, R., NAOR, J., and SCHIEBER, B., “Minimizing service and operation costs of periodic scheduling,” *Mathematics of Operations Research*, vol. 276, pp. 518–544, 2002.
- [5] BITRAN, G. R., HAAS, E. A., and HAX, A. C., “Hierarchical production planning: A single stage system,” *Operations Research*, vol. 29, pp. 717–743, 1981.
- [6] CRAINIC, T. G. and LAPORTE, G., “Planning models for freight transportation,” *European Journal of Operational Research*, vol. 97, pp. 409–438, 1997.
- [7] DASKIN, M. S., *Network and discrete location : models, algorithms, and applications*. New York : Wiley, 1995.
- [8] DERIS, S., OMATU, S., OHTA, H., LT. KUTAR, C. S., and SAMAT, P. A., “Ship maintenance scheduling by genetic algorithm and constraint-based reasoning,” *European Journal of Operational Research*, vol. 112, pp. 489–502, 1999.
- [9] DHALL, S. K. and LIU, C. L., “On a real-time scheduling problem,” *Operations Research*, vol. 26, pp. 127–140, 1978.
- [10] DREZNER, Z., *Facility location : a survey of applications and methods*. New York : Springer, 1995.
- [11] ENGINEER, F. G., NEMHAUSER, G. L., and SAVELSBERGH, M. W. P., “Shortest path based column generation on large networks with many resource constraints,” tech. rep., Industrial and Systems Engineering Department, Georgia Institute of Technology, 2008.
- [12] ESPINOZA, D., GARCIA, R., GOYCOOLEA, M., NEMHAUSER, G. L., and SAVELSBERGH, M. W. P., “Per-seat, on-demand air transportation part i: Problem description and an integer multi-commodity flow model,” *Transportation Science*, vol. 42, pp. 263–278, 2008.



- [13] ESPINOZA, D., GARCIA, R., GOYCOOLEA, M., NEMHAUSER, G. L., and SAVELSBERGH, M. W. P., “Per-seat, on-demand air transportation part ii: Parallel local search,” *Transportation Science*, vol. 42, pp. 279–291, 2008.
- [14] FRANCIS, R. L., MCGINNIS, L. F., and WHITE, J. A., “Locational analysis,” *European Journal of Operational Research*, vol. 12, pp. 220–252, 1983.
- [15] GOETSCHALCKX, M., VIDAL, C. J., and DOGAN, K., “Modeling and design of global logistics systems: A review of integrated strategic and tactical models and design algorithms,” *European Journal of Operational Research*, vol. 143, pp. 1–19, 2002.
- [16] GOPALAN, R. and TALLURI, K. L., “Exact solution of cutting stock problems using column generation and branch-and-bound,” *International Transactions in Operational Research*, vol. 5, pp. 35–44, 1998.
- [17] GRIGORIEV, A., VAN DE KLUNDERT, J., and SPIEKSMAN, F. C. R., “Modeling and solving the periodic maintenance problem,” *European Journal of Operational Research*, vol. 172, pp. 783–797, 2006.
- [18] HAGHANI, A. and SHAFARI, Y., “Bus maintenance systems and maintenance scheduling: model formulations and solutions,” *Transportation Research Part A*, vol. 36, pp. 453–482, 2002.
- [19] HIGLE, J. L. and SEN, S., “Stochastic decomposition: An algorithm for two stage stochastic linear programs with recourse,” *Mathematics of Operations Research*, vol. 16, pp. 650–669, 1991.
- [20] INFANGER, G., *Planning under uncertainty: Solving large scale stochastic linear programs*. Boyd and Fraser Publishing Co., Danvers, MA, 1994.
- [21] KENYON, C., SCHABANEL, N., and YOUNG, N., “Polynomial-time approximation scheme for data broadcast,” *European Journal of Operational Research*, vol. 172, pp. 783–797, 2006.
- [22] KLEYWEGT, A. J., SHAPIRO, A., and HOMEM-DE-MELLO, T., “The sample average approximation method for stochastic discrete optimization,” *SIAM Journal on Optimization*, vol. 12, pp. 479–502, 2001.
- [23] KRALJ, B. and PETROVIC, R., “A multiobjective optimization approach to thermal generating units maintenance scheduling,” *European Journal of Operational Research*, vol. 84, pp. 481–493, 1995.
- [24] MAK, W. K., MORTON, D. P., and WOOD, R. K., “Monte carlo bounding techniques for determining solution quality in stochastic programs,” *Operations Research Letters*, vol. 24, pp. 95–103, 1999.

- [25] MARTELL, D. L., GUNN, E. A., and WEINTRAUB, A., "Forest management challenges for operational researchers," *European Journal of Operational Research*, vol. 104, pp. 1–17, 1998.
- [26] MATTILA, V. and VIRTANEN, K., "A simulation-based optimization model to schedule periodic maintenance of a fleet of aircraft," in *Proc. 2005 European Simulation and Modelling Conference*, (Porto, Portugal), pp. 479–483, 2005.
- [27] MOK, A., ROSIER, L., TULCHINSKY, I., and VARVEL, D., "Algorithms and complexity of the periodic maintenance problem," *Microprocessing and Microprogramming*, vol. 27, pp. 657–664, 1989.
- [28] MUKUNDAN, S. and DASKIN, M. S., "Joint location/sizing maximum profit covering models," *INFOR*, vol. 29, pp. 139–152, 1991.
- [29] NAGY, G. and SALHI, S., "Joint location/sizing maximum profit covering models," *European Journal of Operational Research*, vol. 177, pp. 649–672, 2007.
- [30] NORKIN, V. I., PFLUG, G. C., and RUSZCZYŃSKI, A., "A branch and bound method for stochastic global optimization," *Mathematical Programming*, vol. 83, pp. 425–450, 1998.
- [31] OWEN, S. H. and DASKIN, M. S., "Strategic facility location: A review," *European Journal of Operational Research*, vol. 111, pp. 423–447, 1998.
- [32] PARK, K. S. and YUN, D. K., "Optimal scheduling of periodic activities," *Operations Research*, vol. 33, pp. 690–695, 1984.
- [33] REVELLE, C., D., M., and LIEBMAN, J. C., "An analysis of private and public sector location models," *Management Science*, vol. 16, pp. 692–707, 1970.
- [34] ROMAUCH, M. and HARTL, R. F., *Dynamic Facility Location with Stochastic Demands*. Springer Berlin / Heidelberg, 2005.
- [35] SAAD, G. H., "Hierarchical production-planning systems: Extensions and modifications," *Journal of the Operational Research Society*, vol. 41, pp. 609–624, 1990.
- [36] SANKARAN, J. K. and RAGHAVAN, N. R. S., "Locating and sizing plants for bottling propane in south india," *INTERFACES*, vol. 27, pp. 1–15, 1997.
- [37] SANTOSO, T., AHMED, S., GOETSCHALCKX, M., and SHAPIRO, A., "A stochastic programming approach for supply chain network design under uncertainty," *European Journal of Operational Research*, vol. 167, pp. 96–115, 2005.
- [38] SCOTT, A. J., "Location-allocation systems: A review," *Geographical Analysis*, vol. 2, pp. 95–119, 1970.
- [39] SCOTT, A. J., "Dynamic location-allocation systems: Some basic planning strategies," *Environment and Planning*, vol. 3, pp. 73–82, 1971.

- [40] SNYDER, L. V., "Facility location under uncertainty: a review," *IIE Transactions*, vol. 38, pp. 537–554, 2006.
- [41] SRIRAM, C. and HAGHANI, A., "An optimization model for aircraft maintenance scheduling and re-assignment," *Transportation Research Part A*, vol. 37, pp. 29–48, 2003.
- [42] TAPIERO, C. S., "Transportation-location-allocation problem over time," *Journal of Regional Science*, vol. 3, pp. 377–384, 1971.
- [43] VAN SLYKE, R. M. and WETS, R. J.-B., "L-shaped linear programs with applications to optimal control and stochastic programming," *SIAM Journal on Applied Mathematics*, vol. 17, pp. 638–663, 1969.
- [44] WAGNER, H. M., GIGLIO, R. J., and GLASER, R. G., "Preventive maintenance scheduling by mathematical programming," *Management Science*, vol. 10, pp. 316–334, 1964.
- [45] WEI, W. D. and LIU, C. L., "On a periodic maintenance problem," *Operations Research Letters*, vol. 2, pp. 90–93, 1983.
- [46] WEINTRAUB, A. and CHOLAKY, A., "A hierarchical approach to forest planning," *Forest Science*, vol. 37, pp. 439–460, 1991.
- [47] WEINTRAUB, A., SAEZ, G., and YADLIN, M., "Aggregation procedures in forest management planning using cluster analysis," *Forest Science*, vol. 43, pp. 274–285, 1997.
- [48] WESOLOWSKY, G. O. and TRUSCOTT, W. G., "The multiperiod location-allocation problem with relocation of facilities," *Management Science*, vol. 22, pp. 57–65, 1976.